NEGATIVE CONTRIBUTIONS TO S FROM MAJORANA PARTICLES

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Abstract

In this letter we study oblique corrections from heavy fermions. For certain mass ranges we find negative contributions to the S and T parameters from Majorana particles.

Recently a number of authors [1] - [9] have focused attention on the fact that heavy particles do not necessarily decouple [10] in theories with broken gauge symmetries. If new heavy particles exist and couple only weakly to light fermions, then their effect on low-energy precision electroweak measurements can only arise through contributions to gauge boson self energies. This type of radiative correction is termed an oblique correction [2, 11]. If the masses of the new particles are much larger than the mass of the Z boson, M_Z, then the effects of the oblique corrections can be described by three parameters: S, T, and U [3]. These parameters are defined in terms of the gauge boson self energies:

$$\begin{aligned} S &= 16\pi \frac{d}{dq^2}(\Pi_{33}(q^2) - \Pi_{3Q}(q^2))|_{q^2=0} \quad , \qquad (1) \\ \alpha T &= \Delta\rho_* = \frac{e^2}{s^2 M_W^2}(\Pi_{11}(0) - \Pi_{33}(0)) \quad , \\ U &= 16\pi \frac{d}{dq^2}(\Pi_{11}(q^2) - \Pi_{33}(q^2))|_{q^2=0} \quad , \end{aligned}$$

where we have adopted the notation of Peskin and Takeuchi [3, 5, 11], and M_W is the mass of the W, e is the electromagnetic charge, and $s^2 \equiv \sin^2(\theta_W)$. The indices 1 and 3 refer to $SU(2)_L$ currents, while Q refers to the electromagnetic current. So, for example, in this notation the W self energy is $i(e^2/s^2)\Pi_{11}(q^2)$. The parameter S is sometimes refered to as S_Z, and S_W is defined by $S_W \equiv S + U$. A notation based on chiral Lagrangian conventions uses $L_{10} = -S/(16\pi)$ [3]. (Note that S, T, and U do not include standard model corrections to gauge boson self energies except for contributions from particles much heavier than the Z, eg. the top quark or Higgs boson.) Precision electroweak measurements are beginning to constrain these parameters. While QCD-like technicolor theories predict S and T to be of order 1 [3], and SUSY theories predict S and T to be very small [6], recent global fits to data [7], while not ruling out positive values, give central values of order -1. If experiments continue to favor negative values of S and T while the errors shrink, then it will be useful to have models that can accomodate such values. In this letter we point out that, for this purpose, models using heavy Majorana particles are viable, while those containing only heavy Dirac particles are not. (For an alternative scenario for obtaining negative contributions to S see ref. [8].)

To begin we will discuss the case of heavy degenerate multiplets of Dirac particles. Contributions to S will come from diagrams like those in Fig. 1. By dimensional analysis, such contributions to S must be independent of the fermion mass, since S is dimensionless and there is no other dimensionful parameter aside from the fermion mass. (There is no cutoff dependence since S is finite, see ref. [11].) Thus for the case of degenerate Dirac multiplets, we can discuss contributions from all heavy fermions simultaneously. To simplify the discussion we assemble all the heavy fermion fields into a column vector, and make use of the charge, weak isospin, and hypercharge operators Q, I_3^L, and Y^L. These operators are diagonal matrices with eigenvalues corresponding to the electromagnetic charge, third component of weak isospin, and hypercharge of the left-handed fermions. To be completely general we will consider the possibility that some of the right-handed fields may also belong to arbitrary $SU(2)_L$ multiplets. Thus we will also make use of the operators which give the charges of the right-handed fields: I_3^R and Y^R.

Using the relation

$$Q = I_3^L + \frac{Y^L}{2} = I_3^R + \frac{Y^R}{2} \quad , \tag{2}$$

we find

$$\Pi_{33}(q^2) - \Pi_{3Q}(q^2) = -\frac{1}{2}\Pi_{3Y}(q^2) \quad , \tag{3}$$

so we see that it is sufficient to calculate the (momentum dependent) $W_3 - B$ gauge field mixing (Fig. 1) in order to determine S. Now, the graph in Fig. 1a is proportional to $\mathrm{Tr}(I_3^L Y^L)$, which vanishes within in each multiplet, and the graph in Fig. 1b is proportional to $\mathrm{Tr}(I_3^L Y^R)$. Using Eq. 2 we find

$$\mathrm{Tr}(I_3^L Y^R) = \mathrm{Tr}(I_3^L(2I_3^L - 2I_3^R + Y^L)) = 2\mathrm{Tr}(I_3^L I_3^L - I_3^L I_3^R) \quad . \tag{4}$$

If some of the right handed fields are not singlets, then there will be other contributions to S like those in Fig. 1b, but with L and R interchanged. Thus the full contribution to S from degenerate multiplets of Dirac fermions is

$$S_{\mathrm{Deg.Dirac}} = \frac{1}{3\pi}\mathrm{Tr}(I_3^L I_3^L - 2I_3^L I_3^R + I_3^R I_3^R) = \frac{1}{3\pi}\mathrm{Tr}(I_3^L - I_3^R)^2 \quad , \tag{5}$$

which is positive semi-definite.

We now turn to the case of non-degenerate Dirac particles. For simplicity we will consider the example of a generic doublet $\begin{pmatrix} U \\ D \end{pmatrix}_L$ with right-handed singlets U_R and D_R. Evaluating the graphs in Fig. 1 we find a contribution of the form

$$S_{\rm Dirac} = \frac{1}{6\pi}\left[Y \ln\left(\frac{M_U^2}{M_D^2}\right) + 1\right] \quad . \tag{6}$$

We see that the logarithmic term, which comes from the graph in Fig. 1a, may have either sign depending on the masses and hypercharge of the particles. However, it is well known that (in the absence of flavor-changing neutral currents) the contribution of Dirac particles to T (or $\Delta\rho_*$) is positive semi-definite [12]:

$$T_{\rm Dirac} = \frac{1}{16\pi s^2 M_W^2}\left[M_U^2 + M_D^2 - \frac{2M_U^2 M_D^2}{M_U^2 - M_D^2}\ln\left(\frac{M_U^2}{M_D^2}\right)\right] \quad . \tag{7}$$

Thus, although mass splittings may give rise to negative contributions to S, they also give positive contributions to T. Since experiment favors a negative value for T, and the top quark is also expected to give a positive contribution to T (like that in Eq. 7 multiplied by the number of colors), producing a negative value only for S seems unpromising.

A way around these difficulties is suggested by the result of Bertolini and Sirlin [9]. These authors found that Majorana fermions can give negative contributions to T. Thus we turn to the calculation of S for this case. For completeness we will also present a somewhat simplified discussion of the calculation of T.

We consider a model containing two Majorana (self-conjugate) fields $N = \begin{pmatrix} N_1 \\ N_2 \end{pmatrix}$ with a mass term

$$\mathcal{L}_M = -\frac{1}{2}\overline{N}\begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix} N \quad . \tag{8}$$

The left-handed weak eigenstates are given by [9]

$$\begin{pmatrix} \tilde{N}_L \\ \tilde{N}_R^c \end{pmatrix} = \begin{pmatrix} ic_\theta & s_\theta \\ -is_\theta & c_\theta \end{pmatrix} \frac{1}{2}(1-\gamma_5)N \quad , \tag{9}$$

where $\tilde{N}_R^c \equiv C(\tilde{N}_R)^T$ is a left-handed charge-conjugate field, and c_θ and s_θ denote $\cos(\theta)$ and $\sin(\theta)$, with

$$\begin{aligned} \tan(2\theta) &= \frac{2\sqrt{M_1 M_2}}{M_2 - M_1} \quad , \\ \cos^2(\theta) &= \frac{M_2}{M_1 + M_2} \quad , \\ \sin^2(\theta) &= \frac{M_1}{M_1 + M_2} \quad . \end{aligned} \tag{10}$$

In terms of the $\tilde{N}$ fields, Eq. 8 corresponds to a Dirac mass equal to $\sqrt{M_1 M_2}$ for both left and right-handed weak eigenstates, and a Majorana mass equal to $M_2 - M_1$ for the right-handed fields [9].

We are assuming that the right-handed field $\tilde{N}_R$ is an $SU(2)_L$ singlet, and that the left-handed field is in a doublet with a negatively charged partner, E, with mass M_E, i.e. $\begin{pmatrix} \tilde{N} \\ E \end{pmatrix}_L$. Our results can be easily modified for the case of a positively charged partner, i.e. $\begin{pmatrix} E \\ \tilde{N} \end{pmatrix}_L$. We will not consider any mixing between the new heavy particles and the light generations. With these assumptions (and making use of the self-conjugacy properties of the Majorana fields N), the left-handed neutral current is

$$\overline{\tilde{N}_L}\gamma^\mu \tilde{N}_L = -\frac{c_\theta^2}{2}\overline{N_1}\gamma^\mu\gamma_5 N_1 - \frac{s_\theta^2}{2}\overline{N_2}\gamma^\mu\gamma_5 N_2 + is_\theta c_\theta \overline{N_2}\gamma^\mu N_1 \quad . \tag{11}$$

Similarly, the charged current is

$$\overline{\tilde{N}_L}\gamma^\mu E_L + h.c. = -ic_\theta \overline{N_{1L}}\gamma^\mu E_L + s_\theta \overline{N_{2L}}\gamma^\mu E_L + h.c. \tag{12}$$

We can now calculate the S, T, and U parameters for the case of heavy Majorana fermions. The Feynman rules for Majorana particles are well known [13] and will not be reviewed here. It is convenient to express the gauge boson self energies in terms of vacuum polarizations of left- and right-handed currents. Using an ultra-violet cut-off Λ we have [11]:

$$
\begin{aligned}
&\Pi_{LL}({m_1}^2, {m_2}^2, q^2) = \Pi_{RR}({m_1}^2, {m_2}^2, q^2) = \\
&\quad -\frac{4}{(4\pi)^2} \int_0^1 dx \ln\left[\frac{\Lambda^2}{M^2 - x(1-x)q^2}\right] \left(x(1-x)q^2 - \frac{1}{2}M^2\right),
\end{aligned}
\tag{13}
$$

$$
\begin{aligned}
&\Pi_{LR}({m_1}^2, {m_2}^2, q^2) = \\
&\quad -\frac{4}{(4\pi)^2} \int_0^1 dx \ln\left[\frac{\Lambda^2}{M^2 - x(1-x)q^2}\right] \left(\frac{1}{2}m_1 m_2\right),
\end{aligned}
\tag{14}
$$

where where m_1 and m_2 are the masses of the fermions in the loop, and $M^2 = x m_1^2 + (1-x)m_2^2$. Thus, the graphs in Fig. 1 with the appropriate combinations of mass eigenstates going around the loops can be summarized as:

$$
\begin{aligned}
\Pi_{3Y}(q^2) \quad = \quad & \frac{c_\theta^4}{2}\left[\Pi_{LR}({M_1}^2, {M_1}^2, q^2) - \Pi_{LL}({M_1}^2, {M_1}^2, q^2)\right] \\
& + \frac{s_\theta^4}{2}\left[\Pi_{LR}({M_2}^2, {M_2}^2, q^2) - \Pi_{LL}({M_2}^2, {M_2}^2, q^2)\right] \\
& - s_\theta^2 c_\theta^2\left[\Pi_{LR}({M_1}^2, {M_2}^2, q^2) + \Pi_{LL}({M_1}^2, {M_2}^2, q^2)\right] \\
& + \Pi_{LR}({M_E}^2, {M_E}^2, q^2) + \frac{1}{2}\Pi_{LL}({M_E}^2, {M_E}^2, q^2) \quad .
\end{aligned}
\tag{15}
$$

Using Eqs. 13, 14, and 15 we find the contribution to S is given by

$$
\begin{aligned}
&S_M = \\
&\quad \frac{1}{6\pi}\left\{
\begin{array}{l}
c_\theta^2 \ln\left(\dfrac{M_1^2}{M_E^2}\right) + s_\theta^2 \ln\left(\dfrac{M_2^2}{M_E^2}\right) + \dfrac{3}{2} \\
- s_\theta^2 c_\theta^2 \left[\dfrac{8}{3} + f_1(M_1, M_2) + f_2(M_1, M_2) \ln\left(\dfrac{M_1^2}{M_2^2}\right)\right]
\end{array}
\right\} \quad ,
\end{aligned}
\tag{16}
$$

where

$$f_1(M_1, M_2) = \frac{3M_1M_2^3 + 3M_1^3M_2 - 4M_1^2M_2^2}{\left(M_1^2 - M_2^2\right)^2} \quad , \tag{17}$$
$$f_2(M_1, M_2) = \frac{M_1^6 - 3M_1^4M_2^2 + 6M_1^3M_2^3 - 3M_1^2M_2^4 + M_2^6}{\left(M_1^2 - M_2^2\right)^3} \quad .$$

The calculation of T is summarized by

$$\begin{aligned} \Pi_{11}(0) - \Pi_{33}(0) \quad = \quad & \frac{c_\theta^2}{2}\Pi_{LL}({M_1}^2, {M_E}^2, 0) + \frac{s_\theta^2}{2}\Pi_{LL}({M_2}^2, {M_E}^2, 0) \\ & + \frac{c_\theta^4}{4}\left[\Pi_{LR}({M_1}^2, {M_1}^2, 0) - \Pi_{LL}({M_1}^2, {M_1}^2, 0)\right] \\ & + \frac{s_\theta^4}{4}\left[\Pi_{LR}({M_2}^2, {M_2}^2, 0) - \Pi_{LL}({M_2}^2, {M_2}^2, 0)\right] \\ & - \frac{s_\theta^2 c_\theta^2}{2}\left[\Pi_{LR}({M_1}^2, {M_2}^2, 0) + \Pi_{LL}({M_1}^2, {M_2}^2, 0)\right] \\ & - \frac{1}{2}\Pi_{LL}({M_E}^2, {M_E}^2, 0) \quad . \end{aligned} \tag{18}$$

For the contribution to T we find

$$T_M = \tag{19}$$
$$\frac{1}{16\pi s^2 M_W^2}\left\{\begin{array}{l} c_\theta^2\left[M_1^2 + M_E^2 - \dfrac{2M_1^2M_E^2}{M_1^2 - M_E^2}\ln\left(\dfrac{M_1^2}{M_E^2}\right)\right] \\ + s_\theta^2\left[M_2^2 + M_E^2 - \dfrac{2M_2^2M_E^2}{M_2^2 - M_E^2}\ln\left(\dfrac{M_2^2}{M_E^2}\right)\right] \\ - s_\theta^2c_\theta^2\left[\begin{array}{l} M_1^2 + M_2^2 - 4M_1M_2 \\ \quad + 2\dfrac{M_1^3M_2 - M_1^2M_2^2 + M_1M_2^3}{M_1^2 - M_2^2}\ln\left(\dfrac{M_1^2}{M_2^2}\right) \end{array}\right] \end{array}\right\} .$$

It is not surprising that the contribution to T can be negative, since Eq. 11 contains a mass-eigenstate-changing neutral current.

For completeness we also present the contribution to U:

$$U_M = \frac{1}{6\pi}\left\{ \begin{array}{l} c_\theta^2\left[f_3(M_1,M_E)\ln\left(\frac{M_1^2}{M_E^2}\right)+\frac{4M_1^2M_E^2}{\left(M_1^2-M_E^2\right)^2}\right] \\ +\, s_\theta^2\left[f_3(M_2,M_E)\ln\left(\frac{M_2^2}{M_E^2}\right)+\frac{4M_2^2M_E^2}{\left(M_2^2-M_E^2\right)^2}\right]-\frac{13}{6} \\ +\, s_\theta^2c_\theta^2\left[\frac{8}{3}+f_1(M_1,M_2)-f_2(M_1,M_2)\ln\left(\frac{M_1^2}{M_2^2}\right)\right] \end{array}\right\} , \tag{20}$$

where

$$f_3(M_1,M_E)=\frac{M_1^6-3M_1^4M_E^2-3M_1^2M_E^4+M_E^6}{\left(M_1^2-M_E^2\right)^3} \quad . \tag{21}$$

It can easily be checked that Eqs. 16 and 19 reduce to Eqs. 6 and 7 in the limit that $M_1 \to M_2$.

In order to simplify the discussion of our results we have plotted the curves $S = 0$ and $T = 0$ (lines (a) and (b) respectively) in the M_1, $M_2 - M_1$ plane in Fig. 2. The region where $S < 0$ lies to the left of curve (a), and the region $T < 0$ lies to the right of and above curve (b). The region $U < 0$ is not shown, but it lies entirely in the $T < 0$ region and does not overlap with the $S < 0$ region. Note that the familiar case of Dirac fermions lies along the bottom edge of the graph where $M_1 = M_2$. For the Dirac case we see (as expected) that $S > 0$ for $M_1/M_E > e^{-1/2}$ (where e here is the base of the natural logarithm), and $T \geq 0$ for all values of the masses. (The region $M_2 < M_1$ is not shown in Fig. 2 since there is no region where both S and T are negative.)

It can easily be seen that there exists a region of overlap in Fig. 2 where both S and T are negative, which extends indefinitely in the vertical direction on the graph. It should be noted that while this region of overlap is fairly small, it does not require any unusual values for the masses, i.e. the ratios M_1/M_E and

M_2/M_E can be of order 1. Throughout the overlap region shown in Fig. 2, S is small ($-0.05 < S < 0$). Also shown in Fig. 2 are the curves $\alpha T = \Delta\rho_* = 0.01$ and -0.01 (curves (c) and (d) respectively) assuming $M_E = 500$ GeV. The contribution to T can be made larger by taking M_E to be larger, but it does not make much sense to consider Dirac masses (which break $SU(2)_L \times U(1)_Y$) larger than about 1 TeV. If the Standard Model is correct, then perturbation theory will break down due to the large Yukawa couplings associated with such large masses. If the Standard Model is incorrect then there must be new physics near 1 TeV. This argument should also apply to the Dirac mass of the $\tilde{N}$ fields, so that we should also only consider cases where $\sqrt{M_1 M_2} < 1$ TeV.

To give a idea for the size of S, T, and U in the overlap region, we have calculated them for some interesting vaules of the masses. For $M_1 = 200$ GeV, $M_2 = 2$ TeV, and $M_E = 650$ GeV, we find $S = -1.6 \times 10^{-2}$, $\alpha T = -4.4 \times 10^{-3}$, and $U = 1.8 \times 10^{-2}$. These masses might be considered natural [14] in so far as $\sqrt{M_1 M_2} \approx M_E$. Since the contributions to S and T from the doublet we have considered are typically quite small in the overlap region, it may be necessary to entertain the idea of several doublets in order to obtain contributions of order -1. A more intriguing possibility is that the N and E fields are coupled to some non-Abelian gauge field (eg. technicolor), since then we could naively expect an enhancement by a factor equal to the dimension of the representation to which the fermions belong. The possible gauge groups would be constrained since Majorana particles can only carry real representations. Of course if the gauge coupling is strong (as in technicolor) then there will be non-perturbative corrections that are beyond the scope of this letter.

In our analysis of S and U we have assumed that $M_Z \ll M_1 \leq M_2$ and $M_Z \ll M_E$. That is, the definitions of S and U that we used have neglected momentum dependence in the derivatives of the gauge boson self energies, i.e. (assuming $M_1 < M_E$) we have neglected terms of order M_Z^2/M_1^2. By explicitly calculating the next term in the Taylor series for the self energies we find (taking $M_1 = 200$ GeV) the correction to S,

$$\Delta S = 8\pi M_Z^2 \frac{d^2}{(dq^2)^2}(\Pi_{33}(q^2) - \Pi_{3Q}(q^2))|_{q^2=0} \quad , \tag{22}$$

is between -9×10^{-4} and -4.5×10^{-4} in the overlap region shown in Fig. 2. Since ΔS is negative, the overlap region is slightly larger than shown in Fig. 2.

Given the above discussion one might wonder what the effect on S and T would be if the usual neutrinos of the Standard Model were actually linear combinations of Majorana fermions. This would necessarily introduce a new heavy particle like N_2 discussed above. Of course this scenario requires a more thorough discussion since we could not neglect M_Z compared to M_1 or M_E, however we can see from Eq. 11 that the couplings of the heavy N_2 field vanish like powers of M_1/M_2. In this case, since $M_2 \gg M_1$, a significant contribution can only arise from terms which diverge like powers of M_2. This possibility only occurs in the calculation of T, thus there should be no significant contributions to S and U. Since T depends on the values of self energies at $q^2 = 0$, we can simply take the limiting form from Eq. 19 above. We find:

$$T_M = \frac{1}{16\pi s^2 M_W^2}\left\{ M_E^2 - \frac{2M_1 M_2 M_E^2}{M_2^2 - M_E^2}\ln\left(\frac{M_2^2}{M_E^2}\right)\right\} + \mathcal{O}\left(\frac{M_1^2}{M_W^2}\right) \quad . \tag{23}$$

The terms of order $M_1 M_2/M_W^2$ actually cancel, so we see that in this case the N_2 particle decouples in the usual fashion [10].

In conclusion we have shown that Majorana fermions can give small negative contributions to both S and T for a reasonable range of masses. This may suggest new directions for model building.

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Figure Captions

Fig. 1. Contributions to S from heavy fermions. The wavy lines are $SU(2)_L \times U(1)_Y$ gauge bosons, the solid lines are fermions, and the crosses indicate mass insertions.

Fig. 2. Curves of constant S and constant T. Along curve (a) $S = 0$, and along curve (b) $T = 0$. Assuming $M_E = 500$ GeV, $\alpha T = 0.01$ on curve (c) and -0.01 on curve (d).

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