EXTENDED TECHNICOLOR MODEL BUILDING

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ABSTRACT

We discuss constraints on extended technicolor model building, and show how to rule out classes of models. We also present some speculations on neutrino masses and discuss a toy model of leptons.

1. Introduction

The recent resurgence of technicolor (TC) theories has produced a body of work[1–3] that demonstrates that there are no serious technical obstacles to describing the observed particle mass spectrum (while avoiding flavor changing neutral currents (FCNC's) and keeping ρ close to 1) in terms of a TC model. That is, given the ability to choose an arbitrary TC gauge group with an arbitrary number of technifermions, and representing the extended technicolor (ETC) interactions by four-Fermi interactions with arbitrary mass scales and arbitrary couplings for each of the ordinary fermions one can produce the entire observed range of fermion masses up to a few hundred GeV for the t quark without any phenomenological disasters. Even the S parameter may be held at bay.[4] Though such an exercise is interesting as a sort of existence proof, one worries that since there are more parameters than observables it is impossible to tell whether this success is the result of having the qualitatively correct physics or merely the power of parameter fitting.

To do better we have to be more ambitious and construct models that explain the real world rather than just describe it, i.e. models with fewer parameters than the Standard Model. Such models should make testable predictions, and have the potential to be ruled out by experiment. In principle an explicit ETC model would fit this bill, since it would reduce all the parameters of the Higgs sector to just one parameter for each ETC gauge group. In reality it is unlikely that given the "right" ETC model we would be able to accurately analyze the non-perturbative dynamics involved, so we might be forced to parameterize at least some of our ignorance. Even so, we could still hope to have fewer parameters than observables, and hence a testable model.

In this paper we will discuss some constraints on model building, show how to rule out some classes of models, give some speculations on neutrino masses and

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finish with a discussion of a toy model of leptons.

2. Constraints on Model Building

There are several constraints that one might like to impose on a realistic ETC model. First of all, we expect that there should be more than one ETC scale. The absence of FCNC's (inferred from the $K-\overline{K}$ mass splitting) requires that the ETC scale associated with the the s quark (that is the mass of the gauge boson that connects the s quark to technifermions, and, more importantly, the d) be at least 200 TeV. On the other hand, to obtain a t quark mass of a few hundred GeV (without excessive fine tuning) we expect an ETC scale of order 5 TeV or lower. Such arguments suggest that it is natural to have at least three different ETC scales, one for each family.

Another constraint arises from trying to obtain a large t-b mass splitting, while keeping ρ within 0.5% of 1. To do this without fine tuning it is natural to require that the right-handed t and right-handed b be in different representations of the ETC group ($SU(2)_L$ gauge invariance requires the left-handed t and b to be in the same representation). Having t_R and b_R in different representations allows for the possibility that the t and b are associated with different ETC scales (This scenario has been exploited by King and Mannon and by Einhorn and Nash[3]).

A powerful constraint on ETC model building was originally elucidated by Eichten and Lane,[5] who showed that the absence of a visible axion implies a limit on the number of irreducible representations (irreps) of the ETC gauge group. The number of exactly massless Nambu-Goldstone bosons is given by the number of spontaneously broken global U(1)'s minus the number of global anomalies. For vector-like ETC theories of the form ETC$\otimes SU(2)_L \otimes U(1)$ this implies that there can be at most 3 conjugate pairs of irreps, which implies that quarks and leptons should not be in seperate irreps. Such considerations lead one to consider grand unified technicolor theories (GUTT's) where all interactions (including TC) are unified in a single gauge group. This is the simplest and, in principle, most predictive type of ETC model. We are allowed at most one irrep of the GUTT gauge group, again in order to avoid axions.[5]

3. Ruling Out Classes of Models

Combining the limit on the number of irreps discussed previously with the requirement of three families, we can rule out classes of GUTT's. As we have seen we are allowed only one irrep of the GUTT gauge group. In order to protect ordinary fermions from gaining GUTT scale masses, this single irrep must be complex. Having a complex irrep also allows us to imagine that the GUTT gauge symmetry can be broken down with fermion bilinears, that is without using any scalar fields. However our GUTT must be free of gauge anomalies, so the search can be limited to E_6 and $O(4N+2)$ since these are the only anomaly safe groups with complex representations. The **27** of E_6 and the **64** of $O(14)$ are too small to contain three families, so the smallest possible GUTT is $O(18)$ with all fermions in the 256 dimensional spinor.

This type of theory has been previously discussed[10] as an ordinary GUT, and it may be helpful to review that scenario. If we imagine $O(18)$ breaking to $O(10) \otimes O(8)$ we find that the $\mathbf{256}$ decomposes as $(\mathbf{16}, \mathbf{8}') + (\overline{\mathbf{16}}, \mathbf{8}'')$. The $\mathbf{16}$ of $O(10)$ contains one ordinary family (when $O(10)$ breaks to the Standard Model in the usual fashion) with an extra right handed neutrino, while the $\overline{\mathbf{16}}$ is a family with left and right interchanged. We will refer to the $\overline{\mathbf{16}}$ as a right-family and the $\mathbf{16}$ as a left-family. At this stage the 8 left-families and the 8 right-families are protected from getting masses by gauge symmetries. However, if the $O(8)$ gauge group breaks entirely, then nothing prevents the left-families from getting masses with the right-families, and there will be no light families left over to make up the observed three families.

We can now see what must happen in a GUTT scenario. If the $O(8)$ breaks down to a TC gauge group such that more of the right-families are technicolored than the left-families, then there will be some excess left-families which are protected from getting masses until $SU(2)_L$ is broken. Thus we can proceed straightforwardly by simply listing all the possible TC groups, embedding them (along with the Standard Model) in all possible ways in $O(18)$ and then counting the number of extra techni-singlet left-families†. (This family counting proceedure was originally suggested by Georgi[7] for ordinary GUT's.) In fact we can imagine breaking patterns that do not automatically generate a family structure; in these cases we can simply count quarks and antiquarks. It should be noted that this procedure does not rely on knowing the actual pattern of symmetry breaking between the GUTT scale and the TC scale; different patterns of breaking can result in the same unbroken TC group. The result of this counting procedure is that there is an essentially unique TC group that produces three families from the $O(18)$ GUTT: $Sp(4)_{TC}$. (There exists an embedding of $SU(2)$ into $Sp(4)$ where $\mathbf{4} \to \mathbf{4}$, and $\mathbf{5} \to \mathbf{5}$, but all of our further considerations will apply to this degenerate case as well.) This pattern of breaking was originally discovered by Gell-Mann, Ramond, and Slansky[8] and by Farhi and Susskind.[9] What we have shown is that there are no other possibilities for $O(18)$. In the $Sp(4)_{TC}$ scenario the $\mathbf{8}' \to \mathbf{5} \oplus \mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}$ and the $\mathbf{8}'' \to \mathbf{4} \oplus \overline{\mathbf{4}}$.

Since all of the right-families are technicolored, we expect no super-heavy fermions, that is, except for right-handed Majorana neutrinos, the largest fermion masses are of the order of hundreds of GeV. Above 1 TeV, there is the equivalent of 16 families contributing to beta functions, so asymptotic freedom for QCD and $Sp(4)_{TC}$ is lost (α_s will blow up around 35 TeV). For this theory to make sense, it may be necessary to rely on some non-perturbative dynamics, but computations then become difficult. (For an attempt along these lines, assuming the the existence of nontrivial ultraviolet fixed points, see ref. 11.) One could also consider larger GUTT's like $O(22)$. It is easy to find TC groups which, when embedded in $O(22)$, give four families, but a systematic search for three family models has not been

†The tables of ref. 6 are invaluable in this analysis.

made.

Since the smallest GUTT models seem beyond our computational abilities, it is natural to consider the more general possibility of partial unification, that is where color is unified with TC so that the gauge group is of the form $SU(N)_{ETC} \otimes SU(2)_L \otimes U(1)$. We will consider a fermion content of the form $(\mathbf{R_1}, \mathbf{2}) \oplus (\mathbf{R_2}, \mathbf{1}) \oplus (\mathbf{R_3}, \mathbf{1})$. We will also assume that $SU(3)_C$ is embedded in $SU(N)_{ETC}$ is the simplest possible way (i.e. the $\mathbf{N}$ decomposes to a $\mathbf{3}$ and $N-3$ color singlets) and that $\mathbf{R_1}$, $\mathbf{R_2}$, and $\mathbf{R_3}$ are antisymmetric irreps. These assumptions ensure that the model will only contain $\mathbf{3}$'s and $\overline{\mathbf{3}}$'s of color,[7] that is, we eschew quixes, queights, etc. We also assume that $\mathbf{R_1}$, $\mathbf{R_2}$, and $\mathbf{R_3}$ are not all identical, while requiring that the $SU(N)_{ETC}$ gauge anomaly vanishes. Finally, by counting the number of quarks and antiquarks and requiring three families we can show that such models are ruled out for $N < 10$.

Since such general group theoretical arguments do not get us very far towards a realistic model, we will take a more phenomenological approach in the remainder of the paper.

4. Speculations on Neutrino Masses

The fact that only extremely light left-handed neutrinos are seen poses special problems in TC theories. Fortunately there is a simple explanation available in the usual seesaw mechanism.[8,12] The idea is that right-handed neutrinos get large Majorana masses, so that the left-handed neutrinos end up with masses given by a Dirac mass squared divided by the Majorana mass. Presumably, the neutrino Dirac masses are of the same order as their charged leptonic partners' masses. The remaining problem is to estimate the Majorana masses. One possibility is that the Majorana masses are much larger than any ETC scales, then all the left-handed neutrinos are light, typically with a mass less than 10^{-4} eV for ν_e. This assumes that although there are right-handed techni-neutrinos with ETC interactions, the right-handed neutrinos have no ETC interactions or are exclusively in real representations of the ETC group.

A more interesting possibility[12] is that the Majorana masses are of the same order as some ETC scale (or scales). This is natural if condensates of bilinears of right-handed neutrinos (Majorana condensates) are involved in the dynamical breaking of the ETC gauge symmetry. First we imagine a hierarchy of ETC scales (1000 TeV, 200 TeV, 25 TeV) in order to naturally arrange for a hierarchy of charged lepton masses (m_e, m_μ, m_τ). The 25 TeV scale is a natural scale for producing the mass of the τ, and may also produce the mass of the b when color enhancement effects are included.[13] Now we can estimate the masses of the assocated neutrinos in such a scheme:

$$m_{\nu_e} \approx \frac{(0.5\mathrm{MeV})^2}{1000\mathrm{TeV}} \approx 3 \times 10^{-4}\mathrm{eV} \qquad (1)$$

$$m_{\nu_\mu} \approx \frac{(100\mathrm{MeV})^2}{200\mathrm{TeV}} \approx 50\mathrm{eV} \qquad (2)$$

$$m_{\nu_\tau} \approx \frac{(1\mathrm{GeV})^2}{25\mathrm{TeV}} \approx 40\mathrm{keV}. \tag{3}$$

From these estimates we see three interesting mass scales arising. The ν_e mass is in the right range to impliment the MSW mechanism, provided that there exists another light neutrino for ν_e to mix with. A ν_μ with a mass of the order of 10 eV is interesting as a dark matter candidate. The ν_τ mass is also in an interesting mass range.[14] However if such a heavy Majorana ν_τ has a substantial mixing with the ν_e then another heavy neutrino[15] is required in order to stay below the experimental limit on neutrinoless double β decay. For example, if a 17 keV ν_τ has a 1% mixing with the ν_e, and we use the ν_μ as the second heavy neutrino, then the ν_μ must have either a 17 keV mass, or a mass in the range 150 - 250 keV for consistency with a variety of experiments.[15]

5. A Toy Model

Finally, we will present a toy model of leptons which produces some of the neutrino masses discussed above. As seen in section 2, quarks and leptons should come from the same irrep in order to avoid massless visible axions. Here we postpone quark lepton unification to some scale higher than the ETC scale, so we will be able to discuss leptons seperately. The ETC gauge group in our model is $SU(5)$, which is assumed to commute with the standard model gauge interactions. $SU(5)$ was chosen so that we can break it down to the smallest TC group, $SU(2)_{TC}$, and produce three families. The fermion content of the model is:

$$\nu_{\sigma L} \quad \begin{pmatrix} N & E \\ N & E \\ \nu_\tau & \tau \\ \nu_\mu & \mu \\ \nu_e & e \end{pmatrix}_L \quad \begin{pmatrix} 0 & \nu_e & n & n & N \\ -\nu_e & 0 & n & n & N \\ -n & -n & 0 & \nu_\sigma & \nu_\tau \\ -n & -n & -\nu_\sigma & 0 & \nu_\mu \\ -N & -N & -\nu_\tau & -\nu_\mu & 0 \end{pmatrix}_R \quad \begin{pmatrix} E \\ E \\ \tau \\ \mu \\ e \end{pmatrix}_R \quad . \tag{4}$$

Note that two extra sets of right-handed techni-neutrinos (the n's) and an extra singlet neutrino ($\nu_{\sigma R}$) are needed to fill out the **10**. In general we can expect that the breaking of gauge symmetries at a higher unification scale may leave some sterile particles. Here we assume that the $\nu_{\sigma R}$ has a corresponding (but sterile) $\nu_{\sigma L}$. The most attractive channel for condensation in this model is for the **10** to condense with itself, breaking $SU(5)$ down to $SU(4)$. Each of the **5**'s splits into a **4** and a singlet; these new singlets being the leptons of the first generation. The **10** splits into a **4** and a **6**:

$$\begin{pmatrix} N \\ N \\ \nu_\tau \\ \nu_\mu \end{pmatrix}_R \quad \begin{pmatrix} 0 & \nu_e & n & n \\ -\nu_e & 0 & n & n \\ -n & -n & 0 & \nu_\sigma \\ -n & -n & -\nu_\sigma & 0 \end{pmatrix}_R \quad . \tag{5}$$

The fermions in the **6** obtain Majorana mixing masses of order the breaking scale which we take to be 1000 TeV. Ignoring the heavy **6**, the $SU(4)$ gauge interactions are vector-like. We next imagine that $SU(4)$ is broken down to $SU(3)$ by some as yet unspecified sector of the model (eg. quarks and techniquarks), which splits off the second generation at an assumed scale of 200 TeV. Finally, we assume that the attractive channel (but not the most attractive) for $\nu_{\tau R}$ condensation ($\mathbf{3} \otimes \mathbf{3} \rightarrow \overline{\mathbf{3}}$) gives a Majorana mass of order 25 TeV (the assumed breaking scale) to $\nu_{\tau R}$. This condensation breaks $SU(3)$ down to $SU(2)_{TC}$.

We can now discuss the neutrino mass spectrum of our model. The $\nu_{\sigma L}$ can only receive a mass which is suppressed by scales higher than 1000 TeV so it is approximately massless. The ν_{eL} mass is of order 3×10^{-4} eV as discussed before, so the ν_{eL} and $\nu_{\sigma L}$ are both in the appropriate mass range to allow the MSW mechanism to operate. Also as before, the $\nu_{\tau L}$ mass is of order 40 keV. The case of the ν_μ is more interesting. We have assumed that there is no Majorana mass produced directly, but one may feed down from the $\nu_{\tau R}$ through ETC interactions. The model contains ETC gauge bosons with masses of 200 TeV that connect $\nu_{\mu R}$ with the $\nu_{\tau R}$. The dynamical Majorana mass connects the $\nu_{\tau R}$ with the conjugate $\nu_{\tau R}$, and the same gauge boson takes the $\nu_{\tau R}$ back to a $\nu_{\mu R}$. $SU(3)$ symmetry prevents us from connecting the gauge boson lines, but when $SU(3)$ breaks to $SU(2)_{TC}$ there will in general be some mixing of the order of the breaking scale squared. Thus we can estimate the $\nu_{\mu R}$ Majorana mass by one factor of the condensate (which we approximate by $4\pi(25\mathrm{TeV})^3$) a factor of the gauge boson mixing and four inverse powers of the gauge boson mass:

$$M \approx 4\pi \frac{(25\mathrm{TeV})^5}{(200\mathrm{TeV})^4} \approx 80\mathrm{GeV} \quad . \tag{6}$$

So we find an approximate mass for the left-handed ν_μ given by:

$$m_{\nu_\mu} \approx \frac{(0.1\mathrm{GeV})^2}{80\mathrm{GeV}} \approx 130\mathrm{keV} \quad . \tag{7}$$

Which is close to the region required for the absence of neutrinoless double beta decay (with an assumed 1% $\nu_\tau - \nu_e$ mixing). This model does not contain a dark matter candidate, but the possibility of the ν_μ being heavier than the ν_τ, which at first sight seems implausible, actually arises naturally in the model.

6. Conclusions

We have discussed how some classes of simple ETC models can be ruled out by simply counting quarks. We have made some general speculations on neutrino masses and also presented a toy model of leptons that gives interesting neutrino masses. We have not achieved the goal we set in the introduction, but perhaps these observations will help in the task of constructing a realistic ETC model.

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