Supergravity

Gravity: on-shell

Einstein gravity $\leftrightarrow$ gauge theory of local Lorentz/translation symmetry
generators M_{ab}, P_a $\leftrightarrow$ "gauge fields" ω_μ^{ab}, spin connection, e_μ^a *vierbein*
where $a, b = 0, \ldots, 3$ are Lorentz gauge group indices
$\mu, \nu = 0, \ldots, 3$ are spacetime indices
e_μ^a and ω_μ^{ab} transform as collections of vectors
gauge fields $\leftrightarrow$ feld strengths $R_{\mu\nu}^{ab}$, (Riemann curvature), $C_{\mu\nu}^a$, (torsion)
$C_{\mu\nu}^a = 0$, solve for ω_μ^{ab} in terms of e_μ^a

counting: e_μ^a 16 components
subtract 4 (equations of motion)
subtract 4 (local translation invariance)
subtract 6 (local Lorentz invariance)

leaves 2 degrees of freedom: massless spin-2 particle

Gravity: on-shell

Couplings to matter:

$$\nabla_\mu = \partial_\mu - e_\mu^a P_a - \omega_\mu^{ab} M_{ab}$$

feld strengths can be obtained from

$$\nabla_\mu \nabla_\nu - \nabla_\nu \nabla_\mu$$

Writing

$$e = |\mathrm{det} e_\mu^m|$$

invariant action with only two derivatives is linear in the field strength:

$$S_{\mathrm{GR}} = \frac{M_{\mathrm{Pl}}^2}{2} \int d^4x\, e\, \epsilon^{\mu\nu\rho\lambda} \epsilon_{abcd}\, e_\mu^a e_\nu^b\, R_{\rho\lambda}^{cd} = \frac{M_{\mathrm{Pl}}^2}{2} \int d^4x\, e\, R$$

where R is the curvature scalar

Supergravity: on-shell

$e_\mu^a \leftrightarrow$ helicity 2 particle, $\mathcal{N}=1$ SUSY requires helicity 3/2 ψ_ν^α (gravitino)
on-shell each has two degrees of freedom
gravitino is gauge field $\leftrightarrow Q_\alpha \leftrightarrow$ field strength $D_{\mu\nu\alpha}$
$C_{\mu\nu}^a = 0$, solve for ω_μ^{ab} find on-shell supergravity action:

$$S = \frac{M_{\rm Pl}^2}{2} \int d^4x\, e\, R + \frac{i}{4} \int d^4x\, \epsilon^{\mu\nu\rho\sigma}\, \overline{\psi}_\mu \gamma_5 \gamma_\nu D_{\rho\sigma}$$

call second term $S_{\rm gravitino}$
metric:

$$g_{\mu\nu} = e_\mu^a e_\nu^b \eta_{ab}$$

in terms of a local inertial coordinate system ξ^a at the point X

$$e_\mu^a(X) = \frac{\partial \xi^a}{\partial x^\mu}$$

Brans–Dicke Gravity

first consider toy example, scale-invariant Brans–Dicke theory:

$$S_{\mathrm{BD}} = \int d^4x \, \left[\frac{e}{2} \, \sigma^2 R + \frac{e}{12} \partial^\mu \sigma \partial_\mu \sigma \right]$$

treat scalar σ as a spurion field and set

$$\sigma = M_{\mathrm{Pl}}$$

break local conformal invariance to local Poincaré invariance
$\Rightarrow$ Einstein gravity

Superconformal Gravity

in addition to the “gauge“ fields e_μ^a and $\psi_{\nu\alpha}$ we have $A_\mu \leftrightarrow$ local $U(1)_R$ symmetry, and $b_\mu \leftrightarrow$ local conformal boosts
Counting degrees of freedom off-shell (subtracting gauge invariances):

$$
\begin{array}{rrll}
\text{field} & & & d.o.f. \\
e_\mu^a : & 16 & -4-6-1 & =5 \\
\psi_\nu^\alpha : & 16 & -4-4 & =8 \\
A_\mu : & 4 & -1 & =3 \\
b_\mu : & 4 & -4 & =0
\end{array}
$$

e_μ^a subtract 4 (translation), 6 (Lorentz), 1 (dilations)
ψ_ν^α subtract 4 (SUSY Q_α, $\overline{Q}_{\dot\alpha}$), 4 (conformal SUSY S_β and $\overline{S}_{\dot\beta}$)
A_μ subtract 1(local R-symmetry)
b_μ subtract 4 (four conformal boost generators)
no auxiliary fields for the superconformal graviton multiplet
“gauge” fields, couple with gauge covariant derivatives

Supergravity: off-shell

spurion chiral superfield to break the conformal symmetry:

$$\Sigma = (\sigma, \chi, \mathcal{F}_\Sigma)$$

in global $\mathcal{N} = 1$, Σ is a chiral superfield
here it contains part of the off-shell graviton superfield
Σ called conformal compensator

assign conformal weight 1 to the lowest component of Σ
(x^μ and θ have conformal weight -1 and $-1/2$)
full superconformal gravity action is

$$S_{\rm scg} = \int d^4x \, \frac{e}{2}\sigma^*\sigma R + e\int d^4\,\theta \Sigma^\dagger\Sigma + S_{\rm gravitino}$$

derivatives are covariant in "gauge" fields (e_μ^a, $\psi_{\nu\alpha}$, A_μ, b_μ)
a superconformal Brans–Dicke theory

Supergravity: off-shell

Treat σ, χ, and b_μ as spurion fields

$$\sigma = M_{\rm Pl} \ , \ \ \chi = 0 \ , \ \ b_\mu = 0$$

local superconformal invariance $\rightarrow$ local super-Poincaré invariance
resulting action is:

$$S_{\rm sg} = \int d^4x\, e \left[\frac{M_{\rm Pl}^2}{2} R + \mathcal{F}_\Sigma \mathcal{F}_\Sigma^\dagger - \frac{2M_{\rm Pl}^2}{9} A_\mu A^\mu \right] + S_{\rm gravitino}$$

$\mathcal{F}_\Sigma$ and A_μ are auxiliary fields, counting:

$$\begin{array}{rrlr} \text{field} & & & d.o.f. \\ e_\mu^a : & 16 & -4-6 & = 6 \\ \psi_\nu^\alpha : & 16 & -4 & = 12 \\ A_\mu : & 4 & & = 4 \\ \mathcal{F}_\Sigma : & 2 & & = 2 \end{array}$$

6 bosonic degrees of freedom from $\mathcal{F}_\Sigma$ and A_μ are just what is required to have $\mathcal{N} = 1$ SUSY manifest off-shell

Superspace

eight-dimensional space $z^M = (x^\mu, \theta^\alpha, \overline{\theta}^{\dot{\alpha}})$
require super-general coordinate invariance

$$z^M \to z'^M = z^M + \xi^M$$

where $\xi^M(z^M)$
Superspace scalars transform

$$\phi'(z') = \phi(z)$$

while fields with a superspace index

$$\psi_M = \frac{\partial \phi}{\partial z^M}$$

transform as

$$\psi'_M(z') = \frac{\partial z^N}{\partial z'^M} \psi_N(z)$$

Superspace

construct a vielbein E_M^A
relates the superspace world coordinate to a locally Lorentz covariant (tangent space) coordinate
contains the off-shell multiplet $(e_\mu^a, \psi_{\nu\alpha}, A_\mu, \mathcal{F}_\Sigma)$

we can choose a coordinate system where, for $\theta = 0$,

$$E_\mu^a = e_\mu^a, \;\; E_\mu^\alpha = \frac{1}{2}\psi_\mu^\alpha, \;\; E_\mu^{\dot\alpha} = \frac{1}{2}\overline{\psi}_\mu^{\dot\alpha}$$

Coupling to matter

arbitrary global SUSY theory:

$$\mathcal{L}_{\rm gl} \quad = \quad \int d^4\theta K(\Phi^\dagger, e^V\Phi) + \int d^2\theta \left(W(\Phi) - \tfrac{i\tau}{16\pi} W^\alpha W_\alpha\right) + h.c.$$

define conformal weight 0 fields and mass parameters by

$$\begin{aligned} \Phi' &= \Sigma\,\Phi \\ m' &= \Sigma\, m \end{aligned}$$

dropping the primes, local superconformal-Poincaré invariant Lagrangian:

$$\begin{aligned} \mathcal{L} \quad = \quad & \int d^4\theta f(\Phi^\dagger, e^V\Phi)\tfrac{\Sigma^\dagger\Sigma}{M_{\rm Pl}^2} + \int d^2\theta \tfrac{\Sigma^3}{M_{\rm Pl}^3} W(\Phi) - \int d^2\theta \tfrac{i\tau}{16\pi} W^\alpha W_\alpha + h.c. \\ & -\tfrac{1}{6} f(\phi^\dagger, \phi)\sigma^*\sigma R + \mathcal{F}_\Sigma \mathcal{F}_\Sigma^\dagger - \tfrac{2M_{\rm Pl}^2}{9} A_\mu A^\mu + \mathcal{L}_{\rm gravitino} \end{aligned}$$

action:

$$S \quad = \quad \int d^4x\, e\, \mathcal{L}$$

Coupling to matter

$M_{\rm Pl} \to \infty$ (global SUSY) limit, choose

$$f(\Phi^\dagger, e^V \Phi) = -3\, M_{\rm Pl}^2 e^{-K(\Phi^\dagger, e^V \Phi)/3M_{\rm Pl}^2}$$

rescaling the vierbein by a Weyl (local scale) transformation

$$e_\mu^a \to e^{-K/12M_{\rm Pl}^2} e_\mu^a$$

one finds bosonic piece of the action:

$$\begin{aligned} S_{\rm B} \quad = \quad & \int d^4x\, e \Big[\tfrac{M_{\rm Pl}^2}{2} R + K_j^i(\phi^\dagger, \phi)(\nabla^\mu \phi^i)^\dagger \nabla_\mu \phi_j \\ & -\mathcal{V}(\phi^\dagger, \phi) + \tfrac{i\tau}{16\pi}(F_{\mu\nu}F^{\mu\nu} + iF_{\mu\nu}\widetilde{F}^{\mu\nu}) + h.c. \Big] \end{aligned}$$

where K^i and K_j^i (the Kähler metric) are given by

$$K^i(\phi^\dagger, \phi) = \tfrac{\partial K}{\partial \phi_i} \;, \quad K_j^i(\phi^\dagger, \phi) = \tfrac{\partial^2 K}{\partial \phi^{j\dagger} \partial \phi_i}$$

Coupling to matter

scalar potential:

$$\mathcal{V}(\phi^\dagger, \phi) = e^{K/M_{\rm Pl}^2}\left[\left(K^{-1}\right)_i^j \left(W^i + \frac{W K^i}{M_{\rm Pl}^2}\right)\left(W_j^* + \frac{W^* K_j}{M_{\rm Pl}^2}\right) - \frac{3|W|^2}{M_{\rm Pl}^2}\right] + \frac{g^2}{2}\left(K^i T^a \phi_i\right)^2$$

last term is just the D-term potential

in supergravity the energy density can be negative
usually tune tree-level vacuum energy to zero by adding the appropriate constant to W

Coupling to matter

auxiliary components of chiral superfields (no fermion bilinear VEVs):

$$\mathcal{F}_i = -e^{K/2M_{\rm Pl}^2} \left(K^{-1}\right)_i^j \left(W_j^* + \frac{W^* K_j}{M_{\rm Pl}^2}\right) \qquad (*)$$

from fermionic piece of Lagrangian, $\nabla_\mu \widetilde{\phi}_i$ contains a gravitino term

$$\frac{1}{M_{\rm Pl}} \psi_\mu^\alpha Q_\alpha \widetilde{\phi}_i = \frac{1}{M_{\rm Pl}} \psi_\mu^\alpha \mathcal{F}_i + \mathcal{O}(\sigma^\mu \partial_\mu \phi_i)$$

so the Kähler function contains a term:

$$i K_j^i \frac{1}{M_{\rm Pl}} \bar{\theta} \overline{\widetilde{\phi}}^j \theta^2 \psi_\mu \mathcal{F}_i \sigma^\mu \bar{\theta}$$

in analogy to the ordinary Higgs mechanism, that the gravitino eats the goldstino if there is a nonvanishing $\mathcal{F}$ component

in flat spacetime, goldstino adds right number of degrees of freedom to make a massive spin 3/2 particle

Gravitino Mass

in flat spacetime

$$m_{3/2}^2 = \frac{\mathcal{F}^{*j} K_j^i \mathcal{F}_i}{3M_{\text{Pl}}^2}$$

use (*) and $\mathcal{V} = 0 \Rightarrow$

$$m_{3/2}^2 = e^{K/M_{\text{Pl}}^2} \frac{|W|^2}{M_{\text{Pl}}^4}$$

taking a canonical Kähler function

$$K = Z\Phi^{i\dagger}\Phi_i$$

and $M_{\text{Pl}} \to \infty$ reproduces usual global SUSY results

Maximal Supergravity

massless supermultiplet with helicities ≤ 2
SUSY charges change the helicity by $\frac{1}{2} \Rightarrow \mathcal{N} \leq 8$
arbitrary dimension cannot have more than $32 = 8 \times 4$ real SUSY charges
maximal dimension: spinor in 11 dimensions has 32 components
supergravity theory must have e_μ^a and ψ_μ^α massless gauge fields
D dimensions: "little" group $SO(D-2)$
graviton: symmetric tensor of $SO(D-2)$ has $(D-1)(D-2)/2 - 1$ dof
44 dof for $D = 11$
gravitino is a vector-spinor and a vector has $D-2$ dof
spinor of $SO(D)$ has d_S components, where

$$d_S = 2^{(D-2)/2} \text{ (for } D \text{ even)}, \;\; d_S = 2^{(D-1)/2} \text{ (for } D \text{ odd)}$$

11 dimensions

Majorana spinor has $d_S = 32$ real components, 16 dof on-shell
tracelessness condition $\Gamma^\mu \psi^\alpha_\mu = 0$ leaves $(D-3)d_S/2$ degrees of freedom for the vector-spinor
gravitino has 128 real on-shell dof
gravitino - gaviton = 84 more fermionic dof than bosonic
difference made up by three index antisymmetric tensor $A_{\mu\nu\rho}$
antisymmetric tensor with p indices (i.e. rank p) has

$$\tfrac{1}{p!}\,(D-2)\ldots(D-p-1)$$

dof on-shell, also called a p-form field

$$\tfrac{(11-2)(11-3)(11-4)}{6} = 3 \cdot 4 \cdot 7 = 84$$

11 dimensions: BPS solitons

The SUSY algebra of 11-D supergravity has two central charges
two Lorentz indices, five Lorentz indices $\leftrightarrow$ BPS solitons
central charge acts as a topological charge, spatial integral at fixed t
preserve index structure, solitons extend in two and five spatial directions
called p-branes for p spatial directions
e.g. monopole is a 0-brane, couples to a 1-form gauge field A_μ
p-brane couples to a $(p+1)$-form gauge field
2-brane couples to 3-form gauge field $A_{\mu\nu\rho}$
a p-form gauge field has a $(D-p-2)$-form dual gauge field
field strength $\leftrightarrow$ $A_{\mu\nu\rho}$ is a 4-form: $F_{\mu\nu\rho\lambda}$
contract with ϵ tensor gives dual 7-form $\leftrightarrow$ 6-form dual gauge field
couples to the 5-brane

10 dimensions

compactify 1 dimension on a circle

decompose $D = 11$ fields into massless $D = 10$ fields (constant on circle)

$$
\begin{array}{rcl}
e_\mu^a\,(44) & \rightarrow & e_\mu^a\,(35), B_\mu\,(8), \sigma\,(1) \\
A_{\mu\nu\rho}\,(84) & \rightarrow & A_{\mu\nu\rho}\,(56), A_{\mu\nu}\,(28) \\
\psi_\mu^\alpha\,(128) & \rightarrow & \psi_\mu^{+\alpha}\,(56), \psi_\mu^{-\alpha}\,(56), \lambda^{+\alpha}\,(8), \lambda^{-\alpha}\,(8)
\end{array}
$$

32 supercharges of $D = 11 \rightarrow$ two $D = 10$ spinors

spinors have opposite chirality

gravitino splits into states of opposite chirality, labeled by $+$ and $-$

this is Type IIA supergravity

two other supergravity theories in $D = 10$

Type I: single spinor of supercharges

Type IIB: supercharges are two spinors with the same chirality

Low-Energy Effective Theories

Type IIA $\leftrightarrow$ Type IIA string theory
Type IIB $\leftrightarrow$ Type IIB string theory
Type I with $E_8 \times E_8$ or $SO(32)$ $\leftrightarrow$ heterotic string theory
$D = 11$ supergravity $\leftrightarrow$ M-theory

4D helicities

massless vector $\longrightarrow$ massless 4D vector and $D-4$ massless scalars $\leftrightarrow$ two components with helicity 1 and -1 and $D-4$ helicity 0 states $\Leftrightarrow$ $D-4$ lowering operators
e.g. 5D, the little group is $SO(3)$, one lowering operator $\sigma^- = \frac{1}{2}(\sigma^1 - i\sigma^2)$

traceless symmetric tensor field, e_μ^a, $\Leftrightarrow$ symmetric product of two vectors:

helicity	degeneracy $D=11$	degeneracy $D=10$
2	1	1
1	$7 = 1\cdot(11-4)$	$6 = 1\cdot(10-4)$
0	$28 = 7\cdot 8/2 - 1 + 1$	$21 = 6\cdot 7/2 - 1 + 1$
-1	$7 = 1\cdot(11-4)$	$6 = 1\cdot(10-4)$
-2	1	1

4D helicities: 2-form

2-form field $\Leftrightarrow$ antisymmetric product of two vectors:

helicity	degeneracy $D = 11$	degeneracy $D = 10$
1	7	6
0	$7(7-1)/2 + 1 = 22$	$6(6-1)/2 + 1 = 16$
-1	7	6

where the $7(7-1)/2$ comes from antisymmetrizing the helicity 0 components, and the $+1$ corresponds to combining the helicity 1 and -1 components of the two vectors

4D helicities: 3-form

helicity	degeneracy $D = 11$	degeneracy $D = 10$
1	21	15
0	$35 + 7 = 42$	$20 + 6 = 26$
-1	21	15

35 comes from antisymmetrizing three helicity 0 components, and the $+7$ corresponds to the combining helicity 1 and -1 components and one helicity 0 component of the three vectors

4D helicities: gravitino

$D = 11$ spinor has 8 helicity $\frac{1}{2}$ components and 8 helicity $-\frac{1}{2}$ components, while for $D = 10$ these components correspond to two opposite chirality spinors, we can reconstruct the gravitino by combining a vector and a spinor (remembering the tracelessness condition)

helicity	degeneracy $D = 11$	degeneracy $D = 10$
$\frac{3}{2}$	8	8
$\frac{1}{2}$	$56 = 8 \cdot 7$	$48 = 8 \cdot 6$
$-\frac{1}{2}$	$56 = 8 \cdot 7$	$48 = 8 \cdot 6$
$-\frac{3}{2}$	8	8

$D = 11$ Supermultiplet

starting with a helicity -2 state and raising the helicity repeatedly by acting with 8 SUSY generators (and remembering to antisymmetrize)

11D sugra. state	helicity	degeneracy	e_μ^a	$A_{\mu\nu\rho}$	ψ_μ^α
$\overline{Q}^8 \lvert\Omega_{-2}\rangle$	2	1	1		
$\overline{Q}^7 \lvert\Omega_{-2}\rangle$	$\frac{3}{2}$	8			8
$\overline{Q}^6 \lvert\Omega_{-2}\rangle$	1	28	7	21	
$\overline{Q}^5 \lvert\Omega_{-2}\rangle$	$\frac{1}{2}$	56			56
$\overline{Q}^4 \lvert\Omega_{-2}\rangle$	0	70	28	42	
$\overline{Q}^3 \lvert\Omega_{-2}\rangle$	$-\frac{1}{2}$	56			56
$\overline{Q}^2 \lvert\Omega_{-2}\rangle$	-1	28	7	21	
$\overline{Q} \lvert\Omega_{-2}\rangle$	$-\frac{3}{2}$	8			8
$\lvert\Omega_{-2}\rangle$	-2	1	1		

$D = 10$ Type IIA Supermultiplet

IIA state	helicity	degen.	e^a_μ	$A_{\mu\nu\rho}$	$A_{\mu\nu}$	B_μ	σ	$\psi^{\pm\alpha}_\mu$	$\lambda^{\pm\alpha}$
$\overline{Q}^8 \lvert\Omega_{-2}\rangle$	2	1	1						
$\overline{Q}^7 \lvert\Omega_{-2}\rangle$	$\frac{3}{2}$	8						8	
$\overline{Q}^6 \lvert\Omega_{-2}\rangle$	1	28	6	15	6	1			
$\overline{Q}^5 \lvert\Omega_{-2}\rangle$	$\frac{1}{2}$	56						48	8
$\overline{Q}^4 \lvert\Omega_{-2}\rangle$	0	70	21	26	16	6	1		
$\overline{Q}^3 \lvert\Omega_{-2}\rangle$	$-\frac{1}{2}$	56						48	8
$\overline{Q}^2 \lvert\Omega_{-2}\rangle$	-1	28	6	15	6	1			
$\overline{Q} \lvert\Omega_{-2}\rangle$	$-\frac{3}{2}$	8						8	
$\lvert\Omega_{-2}\rangle$	-2	1	1						

10D: BPS branes

SUSY algebra of Type IIA, central charges of rank 0, 1, 2, 4, 5, 6, 8
$\leftrightarrow$ p-branes
gauge fields of rank 1, 2, and 3
dual gauge fields of rank 5, 6, 7
1-brane $\leftrightarrow$ fundamental string of Type IIA string theory

Type IIA supergravity $\leftrightarrow$ compactified 11D supergravity
2-brane of 11D $\leftrightarrow$ 2-brane of Type IIA when $\perp$ circle
2-brane of 11D $\leftrightarrow$ 1-brane when wraps circle
5-brane of 11D supergravity $\leftrightarrow$ 5-brane and 4-brane of Type IIA

Brane Tensions

11D supergravity has one coupling constant, κ, 11D Newton's constant

$$\mathcal{L} = \frac{1}{2\kappa^2}\, eR$$

11D Planck mass by $\kappa = M_{\rm Pl}^{-9/2}$
tension (energy per unit volume) of branes given powers of $M_{\rm Pl}$
energy per unit area of the 2-brane is $T_2 = M_{\rm Pl}^3$
5-brane we have $T_5 = M_{\rm Pl}^6$
2-brane and 5-brane of Type IIA have same tensions as 11D theory
1-brane and 4-brane have $T_1 = R_{10} M_{\rm Pl}^3$ and $T_4 = R_{10} M_{\rm Pl}^6$
1-brane is the fundamental string of Type IIA string theory
$\Rightarrow$ identify tension with string tension or string mass squared:

$$T_1 = R_{10} M_{\rm Pl}^3 \equiv \frac{1}{4\pi\alpha'} \equiv m_s^2$$

String Coupling

express the tensions in terms of m_s and Type IIA string coupling

$$g_s = (R_{10} M_{\rm Pl})^{3/2}$$

$$T_2 = \frac{m_s^3}{g_s}\ ,\ \ T_4 = \frac{m_s^5}{g_s}\ ,\ \ T_5 = \frac{m_s^6}{g_s^2}$$

branes are nonperturbative BPS solitons not surprising to see inverse powers of the coupling

$1/g_s$ dependence of the 2-brane and 4-brane significant

universal feature of what are now called D-branes

$D = 10$ Type IIB Supermultiplet

SUSY algebra has central charges of rank 1, 3, 5, 7
expect the corresponding p-branes to couple to gauge fields of rank 2 and 4, $A_{\mu\nu}$ and $B_{\mu\nu\rho\lambda}$, and their duals
e_μ^a, ψ_μ^α, λ^α have same dof as the Type IIA, difference being that ψ_μ^α and λ^α have opposite chirality in the IIB theory
it turns out that $A_{\mu\nu}$ is complex, twice as many dof $= 56$
so far the fermions have 37 more dof than the e_μ^a and $A_{\mu\nu}$ combined, while an unconstrained 4-form field has 70 dof
5-form field strength corresponding to $B_{\mu\nu\rho\lambda}$ constrained to be self-dual, reduces dof to 35
need a complex scalar, a, to balance out the multiplet:

$$e_\mu^a\,(35), a\,(2), A_{\mu\nu}\,(56), B_{\mu\nu\rho\lambda}\,(35)$$
$$\psi_\mu^\alpha\,(112), \lambda^\alpha\,(16)$$

$D = 10$ Type IIB Supermultiplet

two SUSY spinor charges of the Type IIB theory have same chirality
transform as vector under an $SO(2)$ group, i.e. R-charges ± 1
single Clifford vacuum state with helicity -2 must have $SO(2)$ charge 0
gravitino splits into two parts with charges ± 1
to antisymmetrize SUSY charges:
antisymmetrize $SO(2)$, symmetrize remaining four spinor indices
or
symmetrize $SO(2)$, antisymmetrize the remaining four spinor indices

$D = 10$ Type IIB Supermultiplet

IIB state	helicity	degeneracy	e_μ^a	$B_{\mu\nu\rho\lambda}$	$A_{\mu\nu}$	a	ψ_μ^α	λ^α
$\overline{Q}^8 \lvert\Omega_{-2}\rangle$	2	1	1					
$\overline{Q}^7 \lvert\Omega_{-2}\rangle$	$\frac{3}{2}$	8					8	
$\overline{Q}^6 \lvert\Omega_{-2}\rangle$	1	28	6	10	12			
$\overline{Q}^5 \lvert\Omega_{-2}\rangle$	$\frac{1}{2}$	56					48	8
$\overline{Q}^4 \lvert\Omega_{-2}\rangle$	0	70	21	15	32	2		
$\overline{Q}^3 \lvert\Omega_{-2}\rangle$	$-\frac{1}{2}$	56					48	8
$\overline{Q}^2 \lvert\Omega_{-2}\rangle$	-1	28	6	10	12			
$\overline{Q} \lvert\Omega_{-2}\rangle$	$-\frac{3}{2}$	8					8	
$\lvert\Omega_{-2}\rangle$	-2	1	1					

$D = 10$ Type IIB Supermultiplet

symmetric combination of 4×4 is 10, antisymmetric under $SO(2)$
$\Rightarrow$ $B_{\mu\nu\rho\lambda}$ has $SO(2)$ charge 0
antisymmetric combination of 4×4 is 6 and graviton has $SO(2)$ charge 0
$\Rightarrow$ two 6's corresponding to $A_{\mu\nu}$ must have charges ± 2.
λ^{α} has $SO(2)$ charge ± 3
scalar a has $SO(2)$ charge ± 4

$D = 10$ Type I Supermultiplet

parity in Type IIB: 4-form, half of 2-form, half of scalar are odd
truncate by keeping only the even fields.
Majorana condition on the fermions reduces dof by one half

$$e_\mu^a\,(35), \sigma\,(1), A_{\mu\nu}\,(28)$$
$$\psi_\mu^\alpha\,(56), \lambda^\alpha\,(8)$$

construction of multiplet has a further complication: only four SUSY raising operators, starting with $|\Omega_{-2}\rangle$ yields a maximum helicity of 0
adding CPT conjugate $\rightarrow$ two helicity 0 components and four helicity $\frac{1}{2}$
graviton requires 21 helicity 0 components
gravitino and spinor require 28 helicity $\frac{1}{2}$ components
need to add 6 copies of a multiplet based on $|\Omega_{-1}\rangle$

$D = 10$ Type I Supermultiplet

Type I state	helicity	degen.	e_μ^a	$A_{\mu\nu}$	σ	ψ_μ^α
$\overline{Q}^4\vert\Omega_0\rangle$	2	1	1			
$\overline{Q}^3\vert\Omega_0\rangle$	$\frac{3}{2}$	4				4
$\overline{Q}^2\vert\Omega_0\rangle + 6\times\overline{Q}^4\vert\Omega_{-1}\rangle$	1	12	6	6		
$\overline{Q}\,\vert\Omega_0\rangle + 6\times\overline{Q}^3\vert\Omega_{-1}\rangle$	$\frac{1}{2}$	28				24
$\overline{Q}^4\vert\Omega_{-2}\rangle + 6\times\overline{Q}^2\vert\Omega_{-1}\rangle + \vert\Omega_0\rangle$	0	38	21	16	1	
$\overline{Q}^3\vert\Omega_{-2}\rangle + 6\times\overline{Q}\,\vert\Omega_{-1}\rangle$	$-\frac{1}{2}$	28				24
$\overline{Q}^2\vert\Omega_{-2}\rangle + 6\times\vert\Omega_{-1}\rangle$	-1	12	6	6		
$\overline{Q}\,\vert\Omega_{-2}\rangle$	$-\frac{3}{2}$	4				4
$\vert\Omega_{-2}\rangle$	-2	1	1			

$D = 10$ Type I and Yang-Mills

$D = 10$, $\mathcal{N} = 1$ (16 real supercharges) Yang-Mills
contains a vector and spinor with eight dof each
couple to Type I supergravity
anomaly cancellation allows for the gauge group $E_8 \times E_8$ or $SO(32)$
two low-energy effective theories for the two heterotic string theories

$D = 10$, $\mathcal{N} = 1$ Yang-Mills is low-energy effective theory for Type I string theory

$D = 5, \mathcal{N} = 8$, gauged supergravity

consider Type IIB supergravity compactified on S^5
integrate out nonzero modes on S^5
$SO(6) \sim SU(4)$ isometry $\rightarrow$ gauge symmetry in the effective theory
5D little group is $SO(3)$
each massless field has one component for each helicity
massless 5D have the same dof as the corresponding massive 4D

graviton has helicities: 2,1,0,-1, and -2: five dof
vector and 2-form have three helicity components 1, 0, and -1
gravitino has four helicity components: 3/2, 1/2, -1/2, and -3/2
spinor has helicity components 1/2 and -1/2

in addition to the $SU(4)$ gauge symmetry, there is $SO(2)$ R-symmetry from Type IIB theory
SUSY generators transform as $(\Box, +1) + (\overline{\Box}, -1)$

5D Graviton Supermultiplet

5D sugra. state	helicity	degeneracy	e_μ^a	A_μ	$B_{\mu\nu}$	ϕ	ψ_μ^α	λ^α
$\overline{Q}^8 \lvert\Omega_{-2}\rangle$	2	1	1					
$\overline{Q}^7 \lvert\Omega_{-2}\rangle$	$\frac{3}{2}$	8					8	
$\overline{Q}^6 \lvert\Omega_{-2}\rangle$	1	28	1	15	12			
$\overline{Q}^5 \lvert\Omega_{-2}\rangle$	$\frac{1}{2}$	56					8	48
$\overline{Q}^4 \lvert\Omega_{-2}\rangle$	0	70	1	15	12	42		
$\overline{Q}^3 \lvert\Omega_{-2}\rangle$	$-\frac{1}{2}$	56					8	48
$\overline{Q}^2 \lvert\Omega_{-2}\rangle$	-1	28	1	15	12			
$\overline{Q} \lvert\Omega_{-2}\rangle$	$-\frac{3}{2}$	8					8	
$\lvert\Omega_{-2}\rangle$	-2	1	1					

5D Graviton Supermultiplet

representations of $SU(4) \times SO(2)$

graviton	e_μ^a	$(1, 0)$
vector	A_μ	$\left(\ydiagram{2,1,1}, 0\right)$
2-form	$B_{\mu\nu}$	$\left(\ydiagram{1,1}, 2\right) + \left(\ydiagram{1,1}, -2\right)$
scalars	ϕ	$(1, \pm 1)$, $\left(\ydiagram{2}, 2\right) + \left(\overline{\ydiagram{2}}, -2\right) + \left(\ydiagram{2,2}, 0\right)$
gravitino	ψ_μ^α	$(\square, 1) + (\overline{\square}, -1)$
“gauginos“	λ^α	$(\square, 3)$, $+ (\overline{\square}, -3) + \left(\ydiagram{2,1}, 1\right) + \left(\overline{\ydiagram{2,1}}, -1\right)$