Gauge mediation

Messengers of SUSY breaking

We will first consider a model with N_f messengers ϕ_i, $\bar{\phi}_i$ Goldstino multiplet X with an expectation value:

$$\langle X \rangle = M + \theta^2 \mathcal{F}$$

$$W = X \bar{\phi}_i \phi_i$$

for gauge unification, ϕ_i and $\bar{\phi}_i$ should form complete GUT multiplets. The existence of the messengers shifts the coupling at the GUT scale

$$\delta\alpha_{\rm GUT}^{-1} = -\frac{N_m}{2\pi} \ln\left(\frac{\mu_{\rm GUT}}{M}\right)$$

where

$$N_m = \sum_{i=1}^{N_f} 2T(r_i)$$

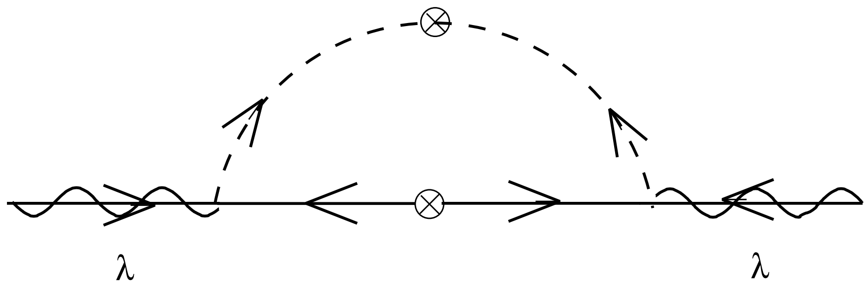
For the unification to remain perturbative we need

$$N_m < \frac{150}{\ln(\mu_{\rm GUT}/M)}$$

Soft Masses

$$\langle X \rangle = M + \theta^2 \mathcal{F}$$

$\Rightarrow$ messenger fermion mass $= M$
$\Rightarrow$ messenger scalars mass^2 $= M^2 \pm \mathcal{F}$



one-loop gaugino mass:

$$M_{\lambda i} \sim \frac{\alpha_i}{4\pi} N_m \frac{\mathcal{F}}{M}$$

Scalar Soft Masses

two-loops squared masses for squarks and sleptons

$$M_s^2 \sim \sum_i \left(\frac{\alpha_i}{4\pi}\frac{\mathcal{F}}{M}\right)^2 \sim M_\lambda^2$$

inserting messenger loop corrections in the one-loop sfermion mass diagrams spoils the cancellation by destroying the relation between the couplings

RG calculation of soft masses

effective Lagrangian below the messenger mass:

$$\mathcal{L}_G = -\frac{i}{16\pi}\int d^2\theta\, \tau(X,\mu) W^\alpha W_\alpha$$

Taylor expanding in the $\mathcal{F}$

$$M_\lambda \;=\; \frac{i}{2\tau}\frac{\partial\tau}{\partial X}\bigg|_{X=M}\mathcal{F} = \frac{i}{2}\frac{\partial\ln\tau}{\partial\ln X}\bigg|_{X=M}\frac{\mathcal{F}}{M}$$

$$\tau(X,\mu) = \tau(\mu_0) + i\frac{b'}{2\pi}\ln\left(\frac{X}{\mu_0}\right) + i\frac{b}{2\pi}\ln\left(\frac{\mu}{X}\right)$$

b' is the β function coefficient including the messengers
b is the β function coefficient in the effective theory (i.e. the MSSM)

$$b' = b - N_m$$

So the gaugino mass is simply given by

$$M_\lambda = \frac{\alpha(\mu)}{4\pi} N_m \frac{\mathcal{F}}{M}$$

Gaugino Masses

$$M_\lambda = \frac{\alpha(\mu)}{4\pi} N_m \frac{\mathcal{F}}{M}$$

the ratio of the gaugino mass to the gauge coupling is universal:

$$\frac{M_{\lambda_1}}{\alpha_1} = \frac{M_{\lambda_2}}{\alpha_2} = \frac{M_{\lambda_3}}{\alpha_3} = N_m \frac{\mathcal{F}}{M}$$

this was once thought to be a signature of gravity mediation models

Sfermion Masses

consider wavefunction renormalization for the matter fields of the MSSM:

$$\mathcal{L} = \int d^4\theta \, Z(X, X^\dagger) Q'^\dagger Q' \ ,$$

Z is real and the superscript $'$ indicates not yet canonically normalized
Taylor expanding in the superspace coordinate θ

$$\mathcal{L} \;=\; \int d^4\theta \left(Z + \frac{\partial Z}{\partial X} \mathcal{F}\theta^2 + \frac{\partial Z}{\partial X^\dagger} \mathcal{F}^\dagger \overline{\theta}^2 + \frac{\partial^2 Z}{\partial X \partial X^\dagger} \mathcal{F}\theta^2 \mathcal{F}^\dagger \overline{\theta}^2 \right) \Big|_{X=M} Q'^\dagger Q'$$

Canonically normalizing:

$$Q = Z^{1/2} \left(1 + \frac{\partial \ln Z}{\partial X} \mathcal{F}\theta^2\right) \Big|_{X=M} Q'$$

so

$$\mathcal{L} = \int d^4\theta \left[1 - \left(\frac{\partial \ln Z}{\partial X} \frac{\partial \ln Z}{\partial X^\dagger} - \frac{1}{Z} \frac{\partial^2 Z}{\partial X \partial X^\dagger} \right) \mathcal{F}\theta^2 \mathcal{F}^\dagger \overline{\theta}^2 \right] \Big|_{X=M} Q^\dagger Q$$

Sfermion Masses

$$m_Q^2 = -\frac{\partial^2 \ln Z}{\partial \ln X \partial \ln X^\dagger}\Big|_{X=M} \frac{\mathcal{F}\mathcal{F}^\dagger}{MM^\dagger}$$

Rescaling also introduces an A term in the effective potential from Taylor expanding the superpotential:

$$W(Q') = W\left(QZ^{-1/2}\left(1 - \frac{\partial \ln Z}{\partial X}\mathcal{F}\theta^2\right)\Big|_{X=M}\right)$$

so the A term is

$$Z^{-1/2}\frac{\partial \ln Z}{\partial X}\Big|_{X=M}\mathcal{F}Q\frac{\partial W}{\partial(Z^{-1/2}Q)}$$

which is suppressed by a Yukawa coupling

RG Calculation

calculate Z and replace M by $\sqrt{XX^\dagger}$, so that $Z(X, X^\dagger)$ is invariant under $X \to e^{i\beta}X$. At l loops RG analysis gives

$$\ln Z = \alpha(\mu_0)^{l-1} f(\alpha(\mu_0)L_0, \alpha(\mu_0)L_X)$$

where

$$L_0 = \ln\left(\frac{\mu^2}{\mu_0^2}\right) \ , \ \ L_X = \ln\left(\frac{\mu^2}{XX^\dagger}\right)$$

so

$$\frac{\partial^2 \ln Z}{\partial \ln X \partial \ln X^\dagger} = \alpha(\mu)^{l+1} h(\alpha(\mu)L_X)$$

two-loop scalar masses are determined by a one-loop RG equation

RG Calculation

at one-loop

$$\frac{d\ln Z}{d\ln\mu} = \frac{C_2(r)}{\pi}\alpha(\mu)$$

$$Z(\mu) = Z_0\left(\frac{\alpha(\mu_0)}{\alpha(X)}\right)^{2C_2(r)/b'}\left(\frac{\alpha(X)}{\alpha(\mu)}\right)^{2C_2(r)/b}$$

where

$$\alpha^{-1}(X) = \alpha^{-1}(\mu_0) + \frac{b'}{4\pi}\ln\left(\frac{XX^\dagger}{\mu_0^2}\right)$$
$$\alpha^{-1}(\mu) = \alpha^{-1}(X) + \frac{b}{4\pi}\ln\left(\frac{\mu^2}{XX^\dagger}\right)$$

So we obtain

$$m_Q^2 = 2C_2(r)\frac{\alpha(\mu)^2}{16\pi^2}N_m\left(\xi^2 + \frac{N_m}{b}(1-\xi^2)\right)\left(\frac{\mathcal{F}}{M}\right)^2$$

where

$$\xi = \frac{1}{1+\frac{b}{2\pi}\alpha(\mu)\ln(M/\mu)}$$

Gauge mediation and the μ problem

electroweak sector in the MSSM needed two types of mass terms: a supersymmetric μ term:

$$W = \mu H_u H_d$$

and a soft SUSY-breaking b term:

$$V = b H_u H_d$$

with a peculiar relation between them

$$b \sim \mu^2$$

In gauge mediated models we need

$$\mu \sim m_{soft} \sim \frac{1}{16\pi^2} \frac{\mathcal{F}}{M}$$

Gauge mediation and the μ problem

If we introduce a coupling of the Higgses to the SUSY breaking field X,

$$W = \lambda X H_u H_d$$

we get

$$\mu = \lambda M \ , \ \ b = \lambda \mathcal{F} \sim 16\pi^2 \mu^2$$

so b is much too large A more indirect coupling

$$W = X(\lambda_1 \phi_1 \bar{\phi}_1 + \lambda_2 \phi_2 \bar{\phi}_2) + \lambda H_u \phi_1 \phi_2 + \bar{\lambda} H_d \bar{\phi}_1 \bar{\phi}_2$$

yields a one-loop correction to the effective Lagrangian:

$$\Delta \mathcal{L} = \int d^4\theta \tfrac{\lambda \bar{\lambda}}{16\pi^2} f(\lambda_1/\lambda_2) H_u H_d \tfrac{X}{X^\dagger}$$

This unfortunately still gives the same, nonviable, ratio for b/μ^2

Gauge mediation and the μ problem

The correct ratio can be arranged with two additional singlet fields:

$$W = S(\lambda_1 H_u H_d + \lambda_2 N^2 + \lambda\phi\bar{\phi} - M_N^2) + X\phi\bar{\phi}$$

then

$$\mu = \lambda_1 \langle S \rangle \ , \ \ b = \lambda_1 \mathcal{F}_S$$

A VEV for S is generated at one-loop

$$\langle S \rangle \sim \frac{1}{16\pi^2} \frac{\mathcal{F}_X^2}{M M_N^2}$$

but $\mathcal{F}_S$ is only generated at two-loops:

$$\mathcal{F}_S \sim \frac{1}{(16\pi^2)^2} \frac{\mathcal{F}_X^2}{M^2} \sim \frac{1}{16\pi^2} \mu \frac{M_N^2}{M}$$

Thus, $b \sim \mu^2$ provided that $M_N^2 \sim \mathcal{F}_X$