16

Electroweak S.B. $\qquad Q = T_3 + Y$

$U(1)_Y$ $\quad D'\big|_{Higgs} = -\frac{g'}{2}\left(|H_u^+|^2 + |H_u^0|^2 - |H_d^0| - |H_d^-|^2\right)$

$SU(2)_W$ $\quad D^a\big|_{Higgs} = -g\left(H_u^* \tau^a H_u + H_d^* \tau^a H_d\right)$

$g' = \frac{e}{\cos\theta_W} \qquad g = \frac{e}{\sin\theta_W}$

$$V = (|\mu|^2 + m_{H_u}^2)(|H_u^0|^2 + |H_u^+|^2) + (|\mu|^2 + m_{H_d}^2)(|H_d^0|^2 + |H_d^-|^2)$$
$$+ b\,(H_u^+ H_d^- - H_u^0 H_d^0) + h.c.$$
$$+ \frac{1}{8}(g^2 + g'^2)\left(|H_u^0|^2 + |H_u^+|^2 - |H_d^0|^2 - |H_d^-|^2\right)^2$$
$$+ \frac{g^2}{2}\left|H_u^+ H_d^{0*} + H_u^0 H_d^{-*}\right|^2$$

assuming $m_{\tilde{q}}^2$ $m_{\tilde{\ell}}^2$ large positive $\langle \tilde{q} \rangle = 0$ $\langle \tilde{\ell} \rangle = 0$

these vevs break E.M., color, B, L

SU(2)$_W$ gauge transformation

$$\langle H_u^+ \rangle = 0$$

$$\frac{\partial V}{\partial H_u^+}\bigg|_{\langle H_u^+ \rangle = 0} = b H_d^- + \frac{g^2}{2} H_d^{0*} H_d^- H_u^{0*}$$

$$\frac{\partial V}{\partial H_u^{+*}}\bigg|_{\langle H_u^+ \rangle = 0} = b^* H_d^{-*} + \frac{g^2}{2} H_u^0 H_d^{-*} H_d^0$$

generically $\langle H_d^- \rangle = 0$

em. is unbroken

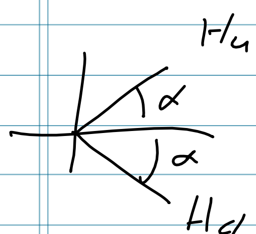
$$V = (|\mu|^2 + m_{H_u}^2)|H_u^0|^2 + (|\mu|^2 + m_{H_d}^2)|H_d^0|^2$$
$$- b H_u^0 H_d^0 + h.c. + \frac{1}{8}(g^2 + g'^2)(|H_u^0|^2 - |H_d^0|^2)^2$$

$$b = |b| e^{i\theta}$$

$$H_u^0 \to e^{i\frac{\theta}{2}} H_u^0 \qquad H_d^0 \to e^{i\theta/2} H_d^0$$

$$\frac{\partial V}{\partial H_u^0} = a H_u^{0*} - b H_d^0$$

$$\langle H_u^{0*} \rangle = \frac{b}{a} \langle H_d^0 \rangle$$



$$U(1)_Y \quad H_u^0 \to e^{-\alpha} H_u^0 \quad H_d \to e^{i\alpha} H_d^0$$

$\langle H_d^0 \rangle, \langle H_d^0 \rangle$ are real

$\Rightarrow$ CP not spont. broken

$\frac{\partial^2 V}{\partial H_i^2}$ negative eigenvalue at origin

$$b^2 > (|\mu|^2 + m_{H_u}^2)(|\mu|^2 + m_{H_d}^2)$$

Stabilize D flat direction

$$b < |\mu|^2 + \frac{m_{H_u}^2 + m_{H_d}^2}{2}$$

$$\langle H_u^0 \rangle = \frac{v_u}{\sqrt{2}} \qquad \langle H_d^0 \rangle = \frac{v_d}{\sqrt{2}}$$

$$\frac{1}{2}(v_u^2 + v_d^2) = \frac{v^2}{2} = \frac{2M_Z^2}{(g^2+g'^2)}$$

$$M_Z^2 = (g^2+g'^2)\frac{v^2}{4} = (g^2+g'^2)\frac{(v_u^2+v_d^2)}{4}$$

$$v = 246 \text{ GeV}$$

$$v_u = v\sin\beta \qquad v_d = v\cos\beta$$

$$\tan\beta = \frac{v_u}{v_d} \qquad \cos 2\beta = \frac{(v_d^2 - v_u^2)}{v^2}$$

$$0 < \beta < \pi/2$$

$$\frac{\partial V}{\partial H_u^0} = 0 \Rightarrow |\mu|^2 + m_{H_u}^2 = b\cot\beta + \frac{M_Z^2}{2}\cos 2\beta$$

$$\frac{\partial V}{\partial H_d^0} = 0 \Rightarrow |\mu|^2 + m_{H_d}^2 = b\tan\beta - \frac{M_Z^2}{2}\cos 2\beta$$

μ-problem

8 real scalars

3 eaten $\pi^0, \pi^\pm$

5

A^0, $H^\pm$, h^0, H^0

CP odd (A^0) CP even (h^0, H^0)

79

$$V \supset (\mathrm{Im}\, H_u^0, \mathrm{Im}\, H_d^0) \begin{pmatrix} b\cot\beta & b \\ b & b\tan\beta \end{pmatrix} \begin{pmatrix} \mathrm{Im}\, H_u^0 \\ \mathrm{Im}\, H_d^0 \end{pmatrix}$$

$$GB \rightarrow \begin{pmatrix} \Pi^0 \\ A^0 \end{pmatrix} = \sqrt{2} \begin{pmatrix} s_\beta & -c_\beta \\ c_\beta & -s_\beta \end{pmatrix} \begin{pmatrix} \mathrm{Im}\, H_u^0 \\ \mathrm{Im}\, H_d^0 \end{pmatrix}$$

$$m_A^2 = \frac{b}{s_\beta c_\beta} \qquad m_W^2 = \frac{g^2 v^2}{4}$$

$$V \supset (H_u^{+*}, H_d^-) \begin{pmatrix} b\cot\beta + M_W^2 c_\beta^2 & b + M_W^2 c_\beta s_\beta \\ b + M_W^2 c_\beta s_\beta & b\tan\beta + M_W^2 s_\beta^2 \end{pmatrix} \begin{pmatrix} H_u^+ \\ H_d^{-*} \end{pmatrix}$$

$$GB \rightarrow \begin{pmatrix} \Pi^+ \\ H^+ \end{pmatrix} = \begin{pmatrix} s_\beta & -c_\beta \\ c_\beta & s_\beta \end{pmatrix} \begin{pmatrix} H_u^+ \\ H_d^{-*} \end{pmatrix}$$

$$m_{H^\pm}^2 = m_A^2 + M_W^2$$

(80)

$$V \supset (\mathrm{Re}\,H_u^0, \mathrm{Re}\,H_d^0) \begin{pmatrix} b\cot\beta + m_Z^2 s_\beta^2 & -b - m_Z^2 c_\beta s_\beta \\ -b - m_Z^2 c_\beta s_\beta & b\tan\beta + m_Z^2 c_\beta^2 \end{pmatrix} \begin{pmatrix} \mathrm{Re}\,H_u^0 \\ \mathrm{Re}\,H_d^0 \end{pmatrix}$$

$$\begin{pmatrix} h^0 \\ H^0 \end{pmatrix} = \sqrt{2} \begin{pmatrix} c_\alpha & -s_\alpha \\ s_\alpha & c_\alpha \end{pmatrix} \begin{pmatrix} \mathrm{Re}\,H_u^0 \\ \mathrm{Re}\,H_d^0 \end{pmatrix}$$

$$\frac{\sin 2\alpha}{\sin 2\beta} - \frac{(m_A^2 + m_Z^2)}{m_{H^0}^2 - m_{h^0}^2} \qquad \frac{\cos 2\alpha}{\cos 2\beta} = -\frac{(m_A^2 - m_Z^2)}{m_{H^0}^2 - m_{h^0}^2}$$

$$m^2_{\binom{H^0}{h^0}} = \frac{1}{2}\left(m_A^2 + m_Z^2 \pm \sqrt{(m_A^2 + m_Z^2)^2 - 4 m_A^2 m_Z^2 \cos^2 2\beta} \right)$$

$m_A,\ m_{H^\pm},\ m_{H^0} \longrightarrow \infty$ as $b \to \infty$

m_{h^0} is maximized at $m_A^2 = \infty$

$$m_{h^0} < |\cos 2\beta|\, m_Z = \frac{(g^2 + g'^2)}{4}(v_d^2 - v_u^2)$$

cf. in SM, $m_{h^0}^2 = \lambda v^2$