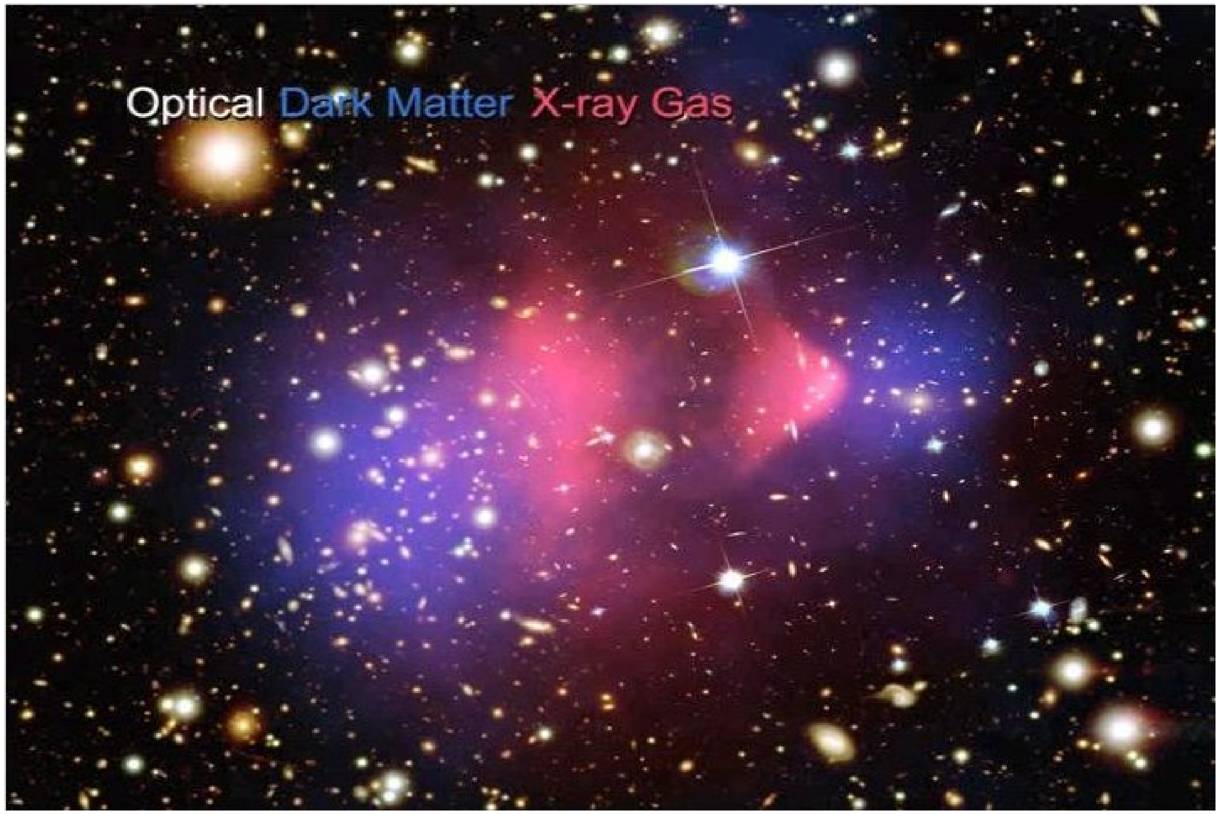
Dark Matter



astro-ph/0608407

Dark Matter Relic Abundance

Robertson-Walker metric and scale factor R

$$ds^2 = -dt^2 + R(t)^2 \left(\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right)$$

Friedman equation

$$H^2 \equiv \left(\frac{\dot{R}}{R} \right)^2 = \frac{8}{3} \pi G \rho - \frac{k}{R^2} + \dots \ ,$$

relates the Hubble parameter H to Newton's constant, G, times the energy density, ρ, the critical density is for $k = 0$ is

$$\rho_c = \frac{3H^2}{8\pi G} \approx 10^{-29} \, \mathrm{g/cm^3} \approx 3 \times 10^{-47} \, \mathrm{GeV^4} \ .$$

Dark Matter Relic Abundance

Energy conservation

$$
\begin{aligned}
R^3 \left(\frac{dp}{dt}\right) &= \frac{d}{dt}\left[R^3\left(\rho + p\right)\right] \\
\frac{dp}{dt} &= -3\frac{\dot{R}}{R}\left(\rho + p\right)
\end{aligned}
$$

for $p = a\rho$

$$\rho \propto R^{-3(1+a)}$$

radiation	$a = 1/3$	$\rho \propto R^{-4}$
matter	$a = 0$	$\rho \propto R^{-3}$
curvature	$a = 0$	$\rho \propto R^{-2}$
vacuum energy	$a = -1$	$\rho \propto R^{0}$

Dark Matter Relic Abundance

a stable weakly interacting dark matter particle X is held in equilibrium by annihilations

$$XX \leftrightarrow p_i\overline{p}_i$$

eventually the expansion of the Universe dilutes the particles so they are too sparse to maintain equilibrium

equilibrium number density, n_{eq}, thermal average of the annihilation cross section times the relative velocity $\langle\sigma v\rangle$

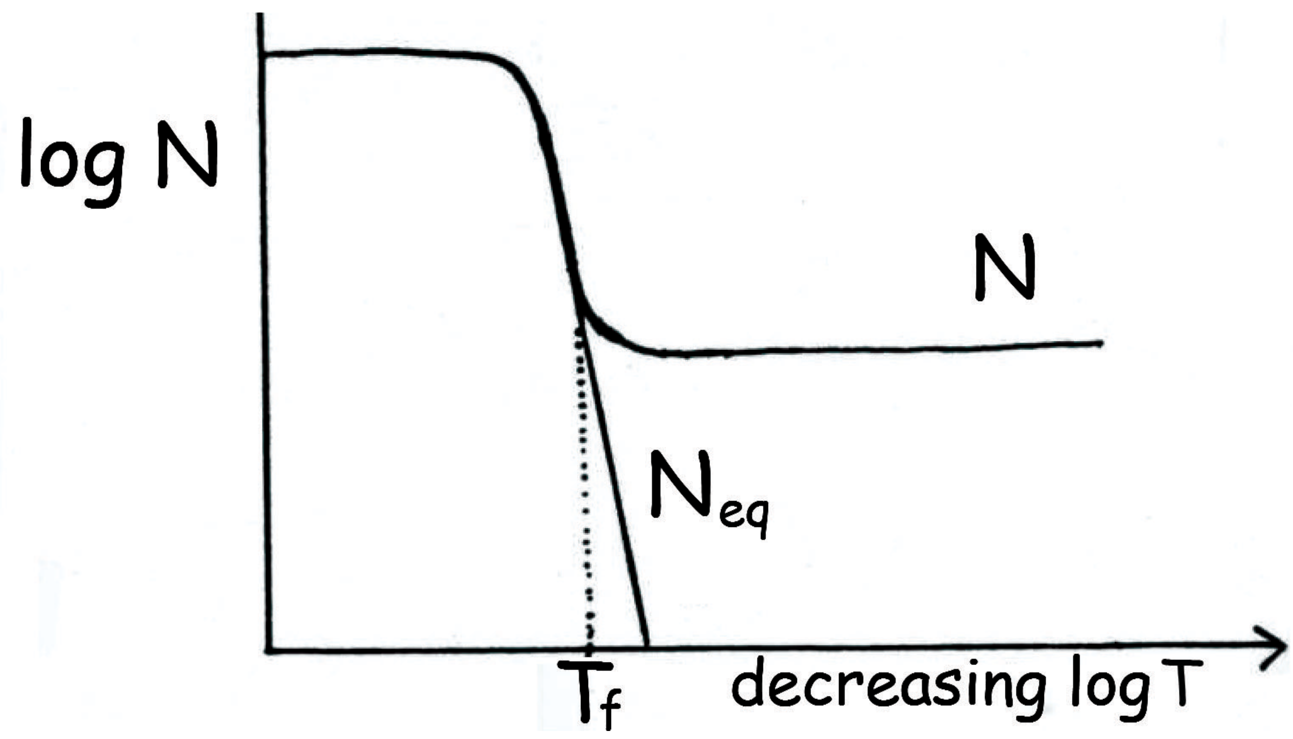
$\dot{n}_{\text{annihilations}} \sim \langle\sigma v\rangle n_{eq}^2$

$\dot{n}_{\text{expansion}} \sim 3Hn_{eq}$

when $\dot{n}_{\text{annihilations}} \approx \dot{n}_{\text{expansion}}$ dark matter 'freezes out"

after freeze out, number of dark matter particles per comoving volume $N \equiv n/T^3$ remains constant

Freeze Out



Quantum Stat. Mech.

Bose-Einstein and Fermi-Dirac

$$\begin{aligned} b(E) &= \frac{1}{e^{(E-\mu)/T}-1} \\ f(E) &= \frac{1}{e^{(E-\mu)/T}+1} \end{aligned}$$

assume chemical potential $\mu = 0$ and relativistic

$$\begin{aligned} N_b &= \frac{g_s}{2\pi^2}\int_0^\infty dp\,\frac{p^2}{e^{p/T}-1} \\ N_f &= \frac{g_s}{2\pi^2}\int_0^\infty dp\,\frac{p^2}{e^{p/T}+1} \end{aligned}$$

scalar	$g_s = 1$
Dirac	$g_s = 2 \times 2 = 4$
Majorana	$g_s = 2$
photon	$g_s = 2$
Z	$g_s = 3$
W	$g_s = 2 \times 3 = 6$

Quantum Stat. Mech.

$$\int_0^\infty dx \frac{x^{\nu-1}}{e^{ax}-1} = a^{-\nu}\,\Gamma(\nu)\zeta(\nu)$$
$$\int_0^\infty dx \frac{x^{\nu-1}}{e^{ax}+1} = (1-2^{1-\nu})\,a^{-\nu}\,\Gamma(\nu)\zeta(\nu)$$

$$N_b = \frac{g_s}{\pi^2}\zeta(3)T^3$$
$$N_f = \frac{3}{4}\frac{g_s}{\pi^2}\zeta(3)T^3$$

$$\rho_b = \frac{g_s}{2\pi^2}\int_0^\infty dp \frac{p^3}{e^{p/T}-1} = \frac{g_s\pi^2}{30}T^4$$
$$\rho_f = \frac{g_s}{2\pi^2}\int_0^\infty dp \frac{p^3}{e^{p/T}+1} = \frac{7}{8}\,\frac{g_s\pi^2}{30}T^4$$

where we used $\zeta(4) = \pi^4/90$

Quantum Stat. Mech.

assume chemical potential $\mu = 0$ and non-relativistic $m \gg T$

$$
\begin{aligned}
N_{f,b} &\approx \frac{g_s}{2\pi^2}\int_0^\infty dp \frac{p^2}{e^{m/T+p^2/(2mT)}\pm 1} \\
&\approx \frac{g_s T^3}{2\pi^2}\int_0^\infty du \frac{u^2}{e^{m/T+u^2 T/m}\pm 1} \\
&\approx \frac{g_s T^3 e^{-m/T}}{2\pi^2}\int_0^\infty du\, u^2\, e^{-u^2 T/m} \\
&\approx \frac{g_s T^3 e^{-m/T}}{(2\pi T/m)^{3/2}}
\end{aligned}
$$

Equilibrium

equilibrium number of nonrelativistic particles per comoving volume:

$$N_{eq} = \frac{e^{-m_X/T}}{(2\pi)^{3/2}} \left(\frac{m_X}{T}\right)^{3/2}$$

above $T \approx 1$ eV the universe is radiation-dominated

$$\rho = \frac{\pi^2}{15} N_* T^4$$

$$N_* = \frac{1}{2} \left(n_b + \frac{7}{8} n_f\right)$$

so

$$H = \sqrt{\frac{8}{3}\pi G\rho} = \sqrt{\frac{8\pi^3 N_* G}{15}}\, T^2$$

$$\langle \sigma v \rangle = \sigma_0 \left(\frac{T}{m}\right)^{\alpha} \quad ,$$

$\alpha = 0$ for Dirac fermion, $\alpha = 1$ for a Majorana fermion

Cross Sections

Dirac fermion:

$$\langle \sigma v \rangle = \frac{G_F^2}{2\pi} m_X^2$$

Majorana fermions have no vector current couplings only axial current:

$$\langle \sigma v \rangle \propto \frac{G_F^2}{2\pi} p^2$$

referred to as p-wave suppression

$$\langle p^2 \rangle = \frac{3}{2} m_X T$$

Freeze Out

Equating the annihilation rate with the expansion rate at $T = T_f$

$$\begin{aligned} \langle \sigma v \rangle n_{eq}^2 &= 3 H n_{eq} \\ \sigma_0 \left(\frac{T_f}{m_X}\right)^{\alpha} \frac{e^{-m_X/T_f}}{(2\pi)^{3/2}} \left(\frac{m_X}{T_f}\right)^{3/2} T_f^3 &= 3\sqrt{\frac{8\pi^3 N_* G}{15}}\, T_f^2 \end{aligned}$$

$$e^{-m_X/T_f} = 3\sqrt{\frac{8\pi^3 N_* G}{15}}\, \frac{(2\pi)^{3/2}}{\sigma_0\, m_X} \left(\frac{m_X}{T_f}\right)^{\alpha - 1/2}$$

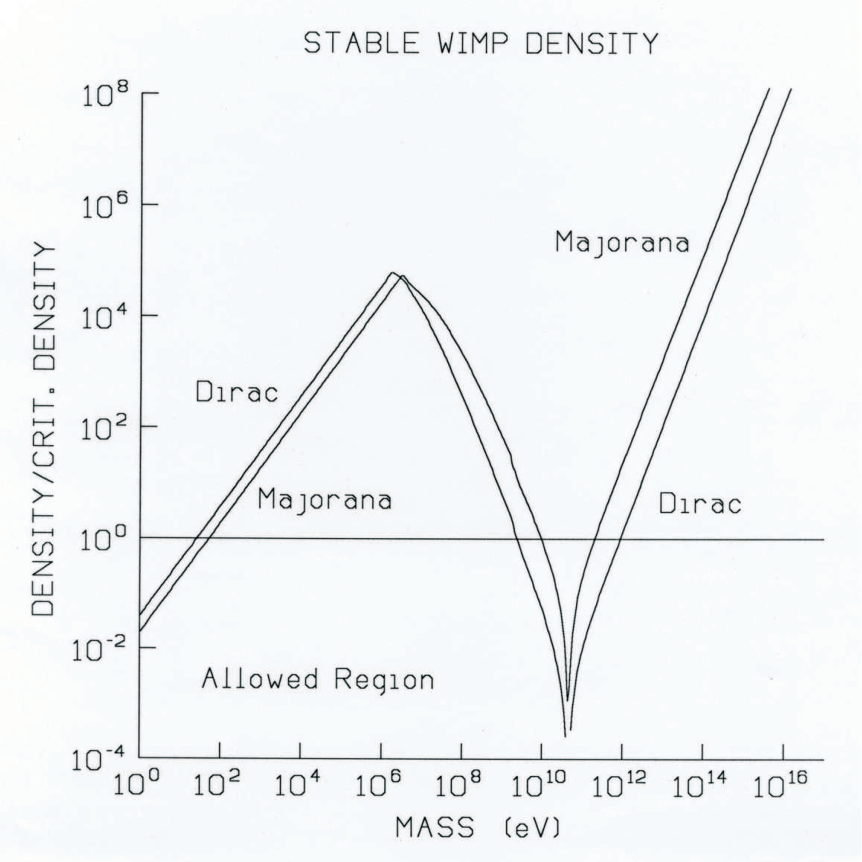
Numerically $m_X/T_f \approx 30$. So the number per comoving volume at T_f is

$$N_f = \sqrt{\frac{8\pi^3 N_* G}{15}}\, \frac{3}{\sigma_0\, m_X} \left(\frac{m_X}{T_f}\right)^{1+\alpha}$$

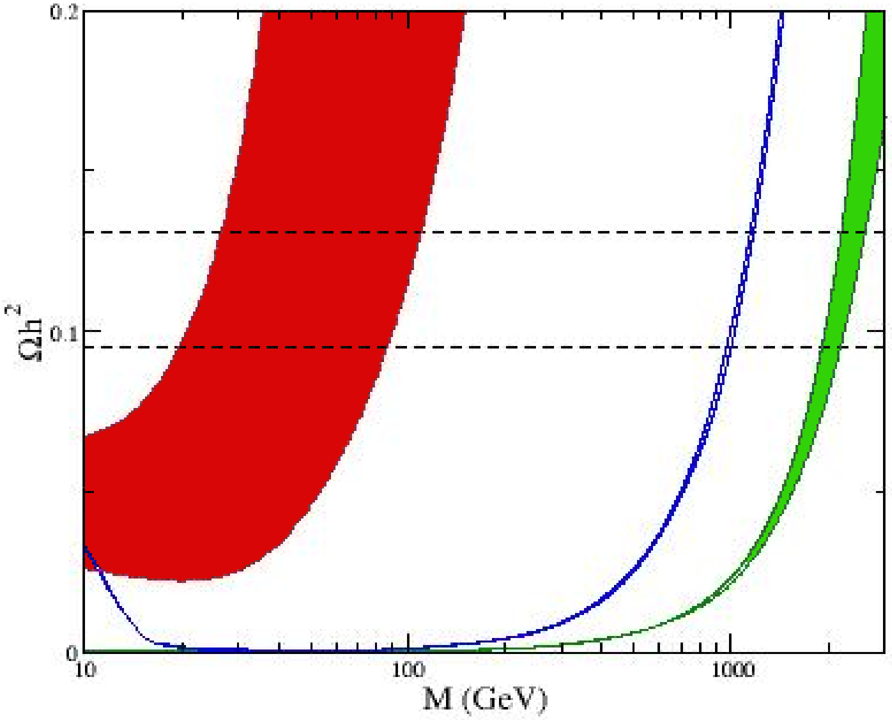
$\times T^3$ gives the number density, $\times m_X$ gives the energy density. weak annihilation cross section $\sigma_0 = N_A G_F^2 m_X^2 / 2\pi$ (where N_A counts final states) with a current temperature of $T = 2.7$ K $= 2 \times 10^{-13}$ GeV, $\alpha = 1$, $N_* = 100$, $N_A = 20$, that

$$\frac{\rho_X}{\rho_c} = 0.6 \left(\frac{100\text{GeV}}{m_X}\right)^2$$

Stable WIMPS



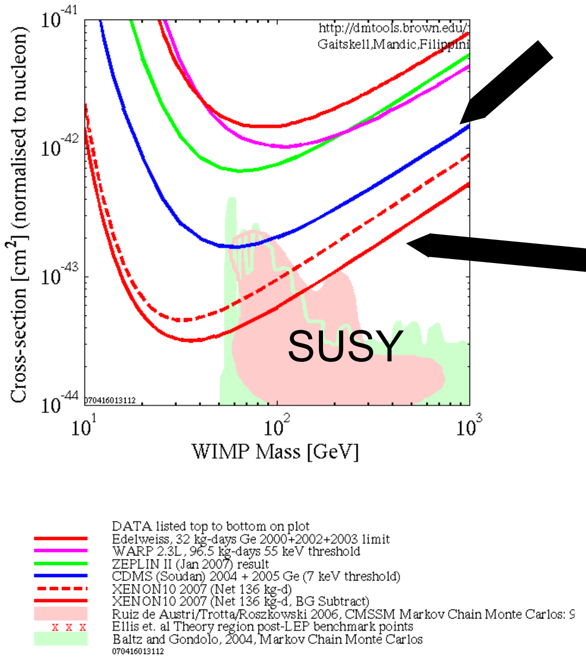
LSP Dark Matter



Bino, Higgsino, Wino

Arkani-Hamed, Delgado, Giudice, hep-ph/0601041

Recent Dramatic Improvement in DM Sensitivity!



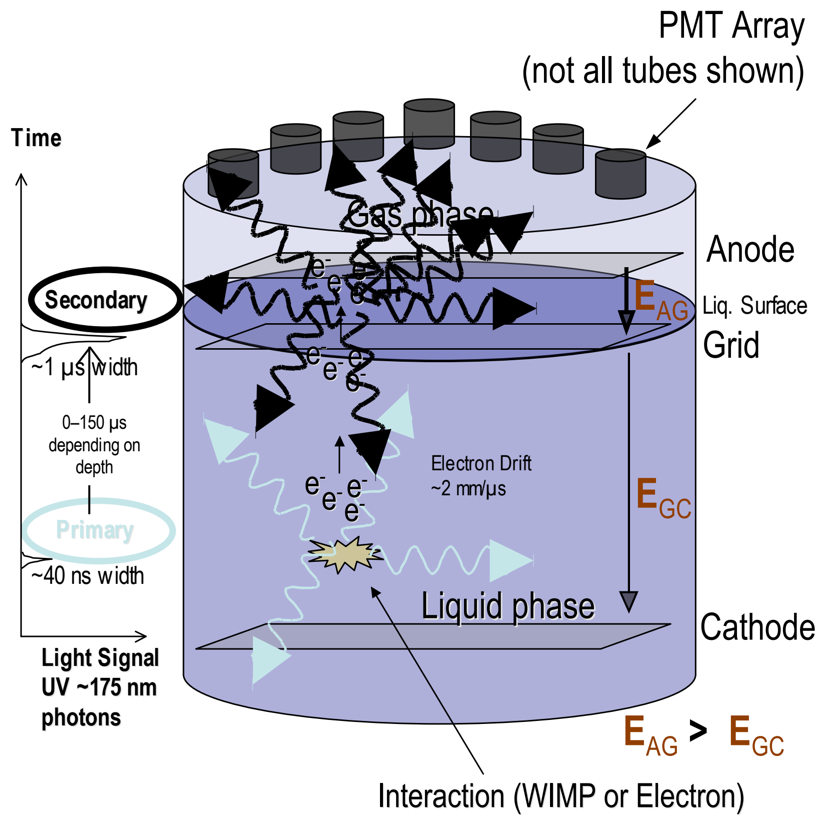
previous best experiment

10 kg fiducial dual phase xenon detector with only 60 days data

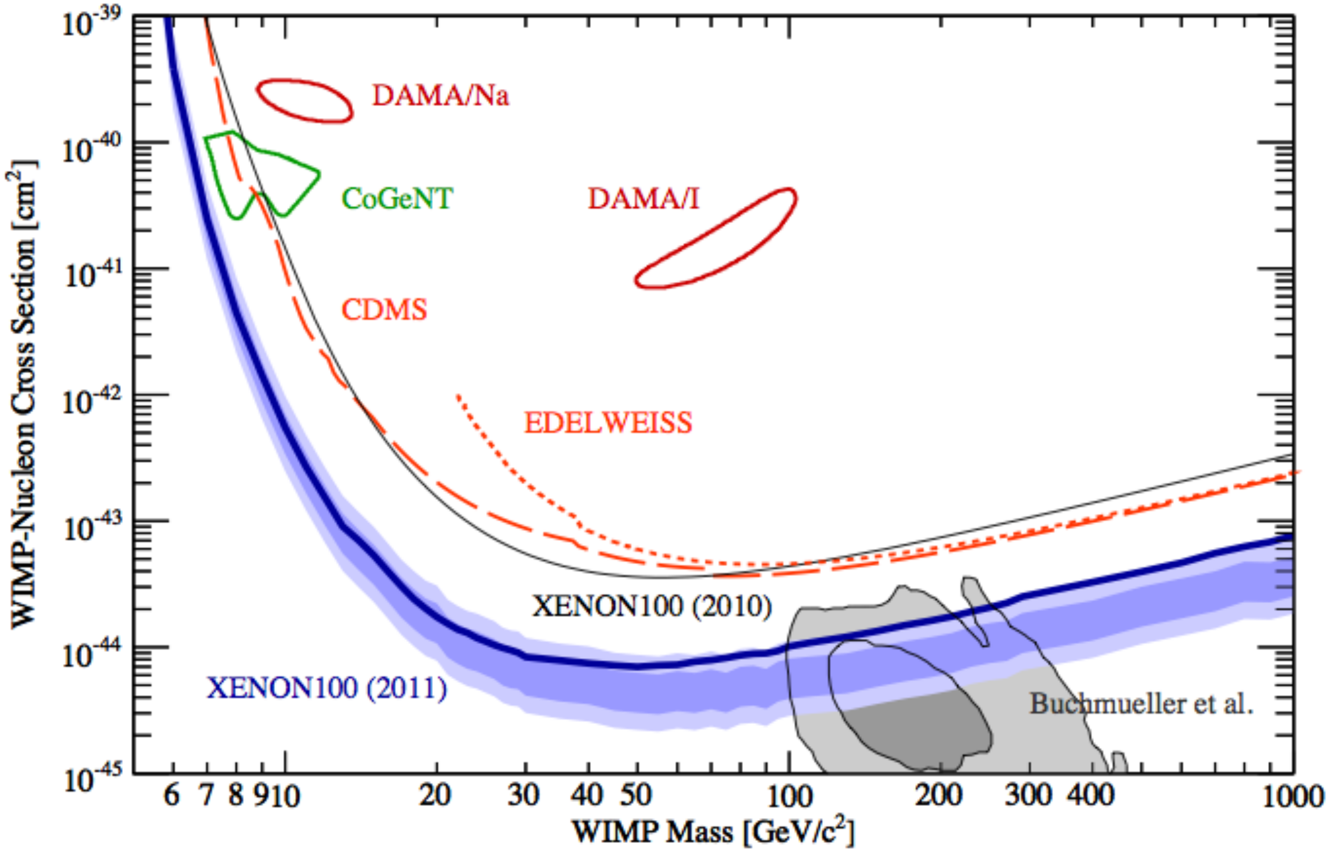
April 17, APS meeting 2008

Assumes local density is not abnormal

Xenon Detector

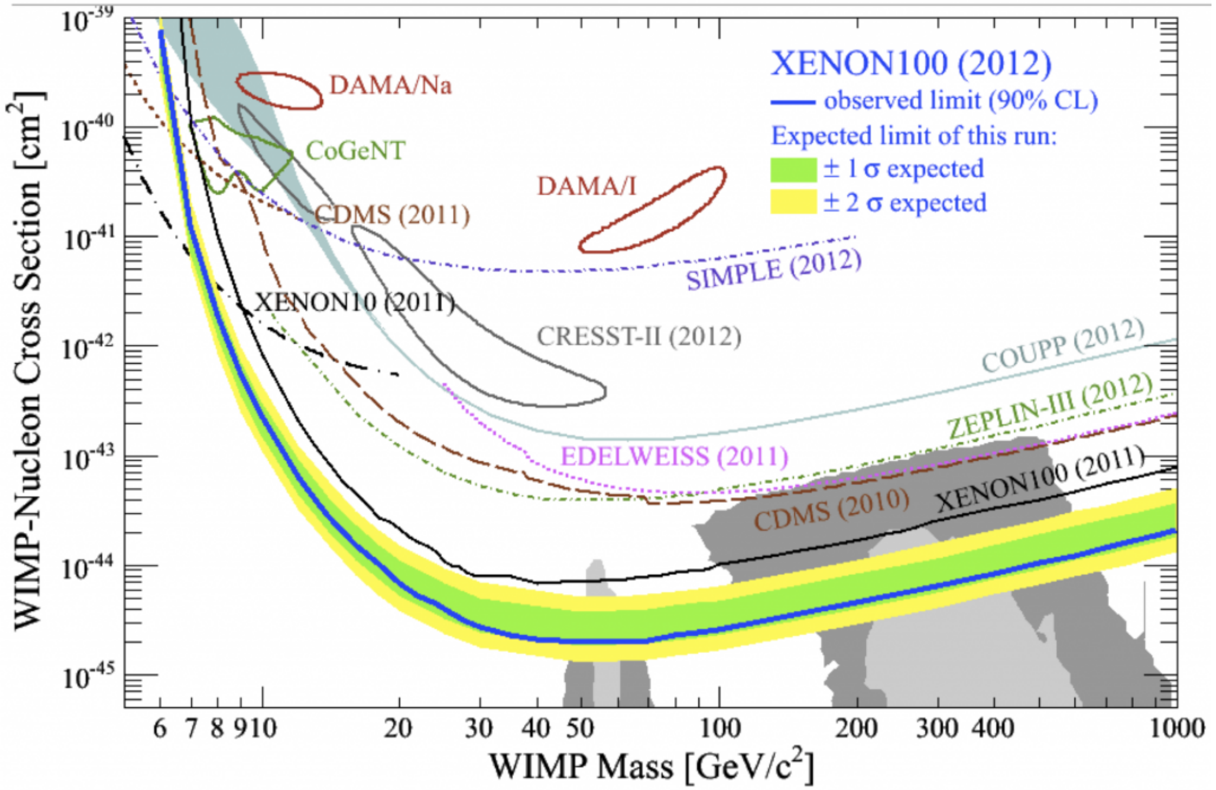


Xenon 100



astro-ph.CO:1104.2549

XENON100: New Spin-Independent Results



Upper Limit (90% C.L.) is 2 x 10^{-45} cm^2 for 55 GeV/c^2 WIMP

astro-ph.CO:arXiv:1207.5988