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Lecture 14

GUTS

$$SU(5) \to SU(3)_c \times SU(2)_L \times U(1)_Y$$

$$5 \to \underset{\uparrow \atop d_R}{(3,1)_{-1/3}} + \underset{(e^+, -\nu^c)_L}{(1,2)_{1/2}}$$

$$\bar{5} \to \underset{\uparrow \atop d_R^c}{(\bar{3},1)_{1/3}} + \underset{(\nu, e)_L}{(1,2)_{-1/2}}$$

$\cancel{5 \times 5}$

$$5 \times \bar{5} \to (1,1)_0 + (8,1)_0 + (3,2)_{-5/6} + (\bar{3},2)_{5/6}$$

$$\parallel \nearrow \quad (1,1)_0 + (1,3)_0$$

$$1 + 24$$

$$24 \to (8,1)_0 + (1,3)_0 + (1,1)_0 + (3,2)_{-5/6} + (\bar{3},2)_{5/6}$$

$$10 = (5 \times 5)_A = \underset{\uparrow \atop u_R^c}{(\bar{3},1)_{-2/3}} + \underset{e^c}{(1,1)_1} + \underset{(u,d)_L}{(3,2)_{1/6}}$$

$\bar{5} + 10$ is anomaly free

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$$G_\mu^a \leftrightarrow T^{1,\dots 8} = \frac{1}{2}\begin{pmatrix} \lambda^{1,\dots 8} & 0 \\ 0 & 0 \end{pmatrix}$$

$$W_\mu^a \leftrightarrow T^{9,10,11} = \frac{1}{2}\begin{pmatrix} 0 & 0 \\ 0 & \sigma^{1,2,3} \end{pmatrix}$$

$$X_\mu, Y_\mu \to T^{12,\dots 23} = \frac{1}{2}\begin{pmatrix} 0 & x \\ x^+ & 0 \end{pmatrix}$$

$$B_\mu \leftrightarrow T^{24} = \frac{1}{2\sqrt{15}}\begin{pmatrix} -2 &&&& \\ & -2 &&& \\ && -2 && \\ &&& 3 & \\ &&&& 3 \end{pmatrix}$$

$$\mathrm{Tr}\, T^{24} T^{24} = \frac{1}{4 \cdot 15}\mathrm{Tr}\begin{pmatrix} 4 &&&& \\ & 4 &&& \\ && 4 && \\ &&& 9 & \\ &&&& 9 \end{pmatrix}$$

$$= \frac{30}{4 \cdot 15} = \frac{1}{2}$$

$$Y = \frac{\sqrt{15}}{3} T^{24} = \sqrt{\frac{5}{3}}\, T^{24}$$

$$g' Y = \left(\sqrt{\frac{5}{3}}\, g'\right)\left(\sqrt{\frac{3}{5}}\, Y\right) = \sqrt{\frac{5}{3}}\, g'\, T^{24} = g_1 T^{24}$$

$$g_1 = \sqrt{\frac{5}{3}}\, g'$$

$$Q_1 = T^{24} = \sqrt{\frac{3}{5}}\, Y$$

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for $SU(5)_{GUT}$

$$g_1 \equiv \sqrt{\frac{5}{3}}\, g' \qquad g_2 \equiv g \qquad g_3 \equiv g_c$$

$$\alpha_i \equiv \frac{g_i^2}{4\pi}$$

measure values at M_Z

$$\alpha_1(M_Z) = 0{,}016830 \pm 0{,}000007$$

$$\alpha_2(M_Z) = 0{,}03347 \pm 0{,}0003$$

$$\alpha_3(M_Z) = 0{,}1187 \pm 0{,}002$$

RG running

$$\mu \frac{dg_i}{d\mu} = \frac{-1}{16\pi^2} b_i g_i^3$$

$$\mu \frac{d\alpha_i^{-1}}{d\mu} = \frac{b_a}{2\pi}$$

In SM and MSSM

$$b_i^{SM} = \left(-\frac{41}{10}, \frac{19}{6}, 7\right)$$

$$b^{MSSM} = \left(-\frac{33}{5}, -1, 3\right)$$

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SM β-functions

$$b_1 = -\frac{2}{3}Q_F^2 - \frac{1}{3}Q_S^2 = -\frac{3}{5}\left(\frac{2}{3}Y_F^2 + \frac{1}{3}Y_S^2\right)$$

$$= -\tfrac{3}{5}\left(\frac{2}{3}N_G\left(3\cdot 2\,Y_Q^2 + 3Y_u^2 + 3Y_d^2 + 2Y_L^2 + Y_e^2\right) + \frac{1}{3}2Y_H^2\right)$$

$$= -\tfrac{1}{5}\left(2N_G\left(6\left(\tfrac{1}{6}\right)^2 + 3\left(\tfrac{2}{3}\right)^2 + 3\left(-\tfrac{1}{3}\right)^2 + 2\left(\tfrac{1}{2}\right)^2 + 1^2\right) + 2\left(\tfrac{1}{2}\right)\right)$$

$$= -\tfrac{1}{5}\left(2N_G\left(\tfrac{1}{6} + \tfrac{4}{3} + \tfrac{1}{3} + \tfrac{1}{2} + 1\right) + \tfrac{1}{2}\right)$$

$$= -\tfrac{1}{5}\left(N_G\frac{(1 + 8 + 2 + 3 + 6)}{3} + \tfrac{1}{2}\right)$$

$$= -\tfrac{1}{5}\left(N_G\,\tfrac{20}{3} + \tfrac{1}{2}\right) = -\frac{41}{10}$$

$$b_2 = \frac{11}{3}\cdot \cancel{2}\,T(A) - \tfrac{2}{3}T(F) - \tfrac{1}{3}T(S)$$

$$= \tfrac{11}{3}\cdot 2 - \tfrac{2}{3}N_G\left(3\cdot\tfrac{1}{2} + \tfrac{1}{2}\right) - \frac{1}{3}\,\tfrac{1}{2}$$

$$= \frac{22}{3} - \frac{4N_G}{3} - \frac{1}{6} = \frac{44}{6} - \frac{24}{6} - \frac{1}{6} = \frac{19}{6}$$

$$b_3 = \frac{11}{3}T(A) - \tfrac{2}{3}T(F) = \frac{11\cdot 3}{3} - \frac{2}{3}N_G\cdot 2\cdot 2\cdot\tfrac{1}{2}$$

$$= \frac{33}{3} - \frac{4}{3}N_G = \frac{33-12}{3} = 7$$

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MSSM β-functions

$$b_1 = -\tfrac{2}{3} q_F^2 - \tfrac{1}{3} Q_S^2 = -Q^2 = -\tfrac{3}{5} Y^2$$

$$= -\tfrac{3}{5}\left(N_G\left(3\cdot 2\cdot Y_Q^2 + 3Y_u^2 + 3Y_d^2 + 2Y_L^2 + Y_e^2\right) + 2\cdot 2 Y_H^2\right)$$

$$= -\frac{3}{5}\left(N_G \frac{20}{6} + 1\right) = \frac{-33}{5}$$

$$b_2 = 3N - F = 3\cdot 2 - N_G\left(3\tfrac{1}{2} + \tfrac{1}{2}\right) - 1$$

$$= 6 - 2N_G - 1 = -1$$

$$b_3 = 3N - F = 3\cdot 3 - 2N_G$$

$$= 9 - 6 = 3$$

couplings converge to

$$3\,\text{GeV} < M_{susy} < 100\,\text{TeV}$$

Radiative Electroweak SB.

$$16\pi^2 \frac{d}{dt} m_{H_u}^2 = 3X_t - 6g_2^2 |M_2|^2 - \frac{6}{5} g_1^2 |M_1|^2$$

$$16\pi^2 \frac{d}{dt} m_{H_d}^2 = 3X_b - X_\tau - 6g_2^2 |M_2|^2 - \frac{6}{5} g_1^2 |M_1|^2 \;)$$

$$16\pi^2 \frac{d}{dt} m_{Q_3}^2 = X_t + X_b - \frac{32}{3} g_3^2 |M_3|^2 - 6g_2^2 |M_2|^2 - \frac{2}{15} g_1^2 |M_1|$$

$$16\pi^2 \frac{d}{dt} m_{\bar{u}_3}^2 = 2X_t - \frac{32}{3} g_3^2 |M_3|^2 - \frac{32}{15} g_1^2 |M_1|^2$$

$$X_t = 2|y_t|^2 (m_{H_u}^2 + m_{Q_3}^2 + m_{\bar{u}_3}^2) + 2|a_t|^2$$

$$X_b = 2|y_b|^2 (m_{H_d}^2 + m_{Q_3}^2 + m_{\bar{d}_3}^2) + 2|a_b|^2$$

$$X_\tau = 2|y_\tau|^2 (m_{H_d} + m_{L_3}^2 + m_{\bar{e}_3}) + 2|a_\tau|^2$$

gluino masses drive squarkmasse ↑ in IR

gauge terms are additive

Keep: $|y_t|^2$ terms neglect y_t running

(quasi fixed point

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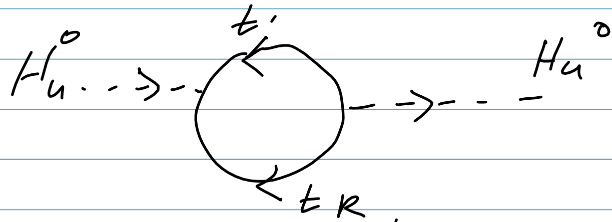
$$\frac{d}{dt} m_{H_d}^2 = 0$$

$$16\pi^2 \frac{d}{dt} \begin{pmatrix} m_{H_u}^2 \\ m_{\bar{u}_3}^2 \\ m_{Q_3}^2 \end{pmatrix} = 2|y_t|^2 \begin{pmatrix} 3 & 3 & 3 \\ 2 & 2 & 2 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} m_{H_u}^2 \\ m_{\bar{u}_3}^2 \\ m_{Q_3}^2 \end{pmatrix}$$



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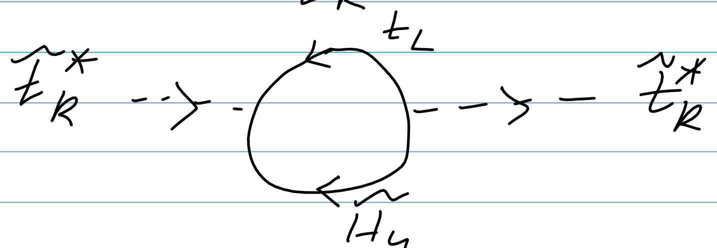

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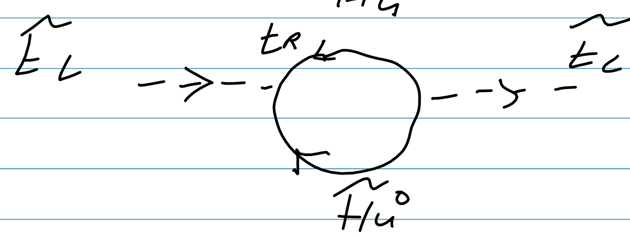

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eigenvalues: $0, 0, 6$ eigenvectors $\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$

$$\begin{pmatrix} m_{H_u}^2 \\ m_{\bar{u}_3}^2 \\ m_{Q_3}^2 \end{pmatrix} = m_0^2 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \frac{1}{2} m_0^2 \left(- \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ +1 \\ -1 \end{pmatrix} + \underbrace{\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}}_{\text{runs to } 0} \right)$$

$$\Big\downarrow \text{IR}$$

$$= \frac{1}{2} m_0^2 \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

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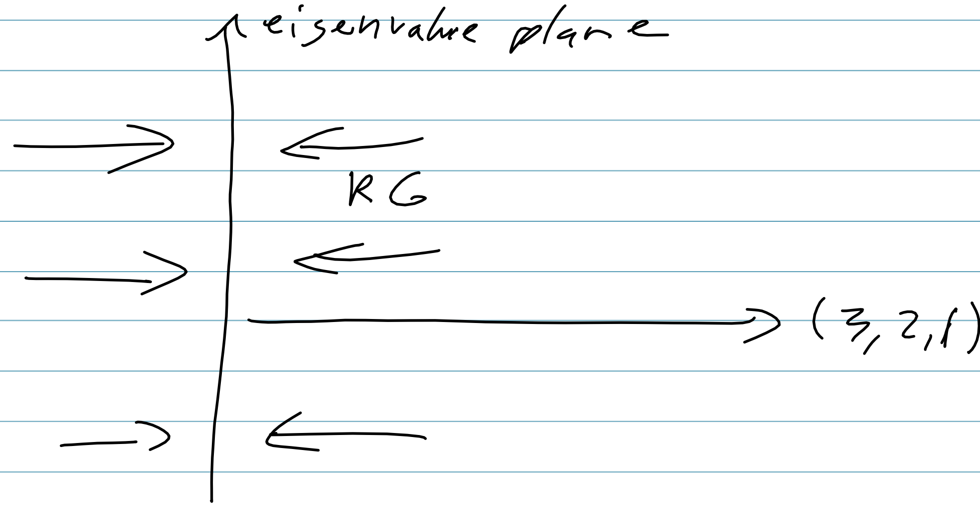
$t = \ln \mu$

$$\frac{dm^2}{dt} = a m^2$$

$$\frac{dm^2}{m^2} = a\,dt$$

$$\ln m^2 = at + c = a \ln \mu + c$$

$$m^2 = c' \left(\frac{\mu}{\mu'}\right)^a$$



$m_{H_u}^2$ runs negative

→ radiative electroweak SB (if μ and b are chosen correctly)

usually claim that this calculation "predicted" large m_t

but only required

$$y_t = \frac{\sqrt{2}\, m_t}{v \sin\beta} \quad \text{large}$$