SUSY Breaking

$\langle 0|H|0\rangle > 0$ $\quad V = F^{i*}F_i + \frac{1}{2}D^a D^a$

O' Raifeartaigh: no simultaneous solutions $\frac{\delta W}{\delta \phi_i} = 0$

$$W = -k^2\phi_1 + m\phi_2\phi_3 + \frac{y}{2}\phi_1\phi_3^2$$

$$F_1^* = \frac{\delta W}{\delta \phi_1} = -k^2 + \frac{y}{2}\phi_3^2 \Rightarrow \phi_3 \neq 0$$

$$F_2^* = \frac{\delta W}{\delta \phi_2} = m\phi_3 \Rightarrow \phi_3 = 0, \quad F_3^* = \frac{\delta W}{\delta \phi_3} = m\phi_2 + y\phi_1\phi_3$$

$$V = |F_1|^2 + |F_2|^2 + |F_3|^2 > 0$$

global minimum at $\phi_2 = \phi_3 = 0$, ϕ_1 undetermined

$V = |F_1|^2 = k^4$ $\quad$ F-flat direction

$$V = \left|-k^2 + \frac{y}{2}\phi_3^2\right|^2 + |m\phi_3|^2 + |m\phi_2 + y\phi_1\phi_3|^2$$

$$= k^4 - \frac{ky}{2}(\phi_3^2 + \phi_3^{*2}) + m|\phi_3|^2 + m|\phi_2|^2 + \text{interaction}$$

$$\phi_3 = \frac{1}{\sqrt{2}}(a + ib) \qquad \phi_3^2 = \frac{1}{2}(a^2 + 2aib - b^2)$$

$$\phi_3^2 + \phi_3^{*2} = a^2 - b^2$$

$$V = k^4 - \frac{k^2 y}{2}(a^2 - b^2) + \frac{m^2}{2}(a^2 + b^2) + m|\phi_2|^2 + \dots$$

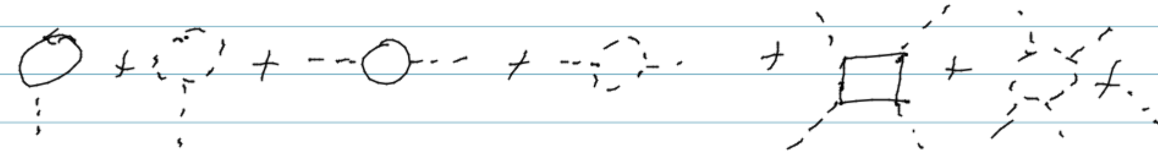
Scalar spectrum $0, 0, m^2 - k^2 y, m^2 + k^2 y, m^2, m^2$

fermions: $-\frac{1}{2}(\psi_1\ \psi_2\ \psi_3)\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & m \\ 0 & m & 0 \end{pmatrix}\begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix}$ $\quad 0 = \begin{vmatrix} -\lambda & 0 & 0 \\ 0 & -\lambda & m \\ 0 & m & -\lambda \end{vmatrix} = -\lambda(\lambda^2 - m^2)$

fermion spectrum $0, m, m$

since SUSY is broken
quantum corrections will lift flat direction

V_{CW} =

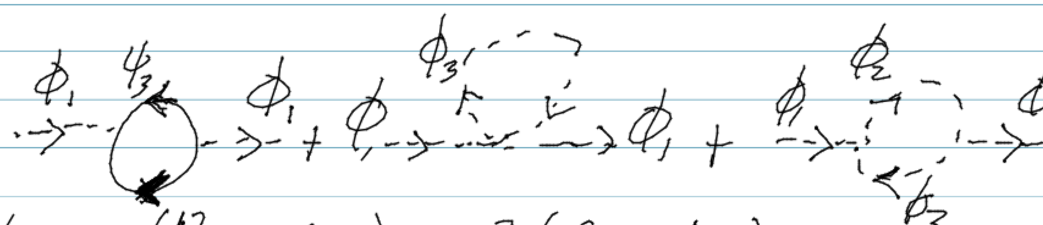


$m_{\phi_1}^2 > 0$ min at $\phi_1 = 0$

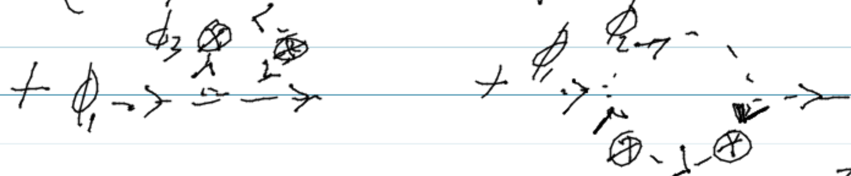
ψ_1 fermion remains massless → goldstino
partner of F_1

$$V = k^4 - \frac{k^2 y}{2}(\phi_3^2 + \phi_3^{*2}) + m^2|\phi_3|^2 + m^2|\phi_2|^2$$
$$+ ym(\phi_2^*\phi_1\phi_3 + \phi_2\phi_1^*\phi_3^*) + y^2|\phi_1|^2|\phi_3|^2 + \frac{y^2}{4}|\phi_3^2$$

$-iM^2_{\text{one-loop}}$ =



$$\left(-y^2(\Lambda^2 + m^2\ln\Lambda) + y^2(\Lambda^2 + m^2\ln\Lambda) + (ym)^2\ln\Lambda\right)$$



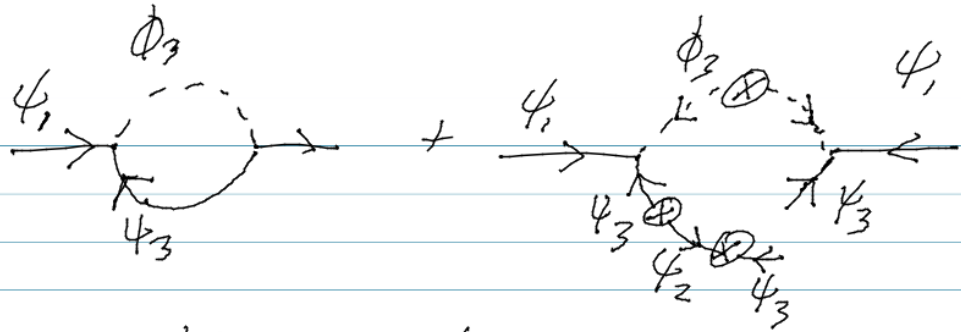
$$-im_1^2 = \int \frac{d^4p}{(2\pi)^4}(-iy^2)\frac{i^3(ik^2y)^2}{(p^2-m^2)^3} + (-iym)^2\frac{i}{p^2-m^2}\frac{ik^4y^2}{(p^2-m^2)^3}$$

$$= -y^4k^4\int_0^\infty \frac{ip^2dp^2}{16\pi^2}\frac{1}{(p^2+m^2)^3}\left(\frac{p^2+m^2-m^2}{(p^2+m^2)}\right)$$

$$= -i\frac{k^4y^4}{16\pi^2}\int_{m^2}^\infty dx\,\frac{(x-m^2)^2}{x^4} = -i\frac{k^4y^4}{16\pi^2}\left[\frac{-1}{x}+\frac{m^2}{x^2}-\frac{m^4}{3x^3}\right]_{m^2}^{\infty}$$

$$= -i\frac{k^4y^4}{16\pi^2 m^2}\left(1-1+\frac{1}{3}\right) = -i\frac{k^4y^4}{48\pi^2m^2}$$

$$\boxed{m_1^2 = \frac{y^4}{48\pi^2}\frac{|F|^2}{m^2}}$$



goldstino!

Fayet-Iliopoulos D-term breaking

for U(1) gauge theory, add

$\mathcal{L}_{FI} = K^2 D$ $\qquad \delta D \to$ total derivative

$V = \frac{1}{2} D^2 - K^2 D + g D q_i \phi^{*i} \phi_i$

eq. of motion $D = K^2 - g q_i \phi^{*i} \phi_i$

add a superpotential that forces $\langle \phi_i \rangle = 0$

then $D = K^2$

won't work $U(1)_Y$ in MSSM

no quadratic susy terms for squarks, sleptons

=> bad revs

adding new U(1) hard to get all the masses
MSSM

Fayet-Illiopoulos $\sqrt{D} \sim K \to$ bad spectrum
O'Raifeartaigh $\sqrt{F} \sim k$ no gauge singlet to get $F \neq 0$
$Y, m \to 0$ still breaks SUSY

SUSY breaking scale put in by hand

to get SUSY breaking naturally $\ll M_{pl}$
need an asymptotically free gauge theory

$$\Lambda = e^{-\frac{8\pi^2}{b g_0^2}} M_{pl}$$

need to understand strong dynamics at scale Λ
(duality)

typically find $F_{\text{composite operator}} \neq 0$

So MSSM needs to be extended

since F_{comp} can't correspond to any MSSM supermultiplet

F_{comp} can't couple through tree-level renormalizable operators
and gaugino masses $\to \phi \lambda$

$$\mathrm{Tr}\, M^2_{\text{real scalar}} = 2\,\mathrm{Tr}\, M^2_{\text{fermions}}$$

still holds with tree-level breaking
$\to$ light superpartners

also why are we in a safe neighborhood?

Scenario:

$\boxed{\begin{matrix}\text{DSB}\\ \text{(hidden)}\end{matrix}}$ ~~~~ flavor blind ~~~~ $\boxed{\begin{matrix}\text{MSSM}\\ \text{(visible)}\end{matrix}}$

"gravity mediation"

$$m_{soft} = \frac{\langle F \rangle}{M_{pl}} \quad \to \text{as } F \to 0, \; M_{pl} \to \infty$$

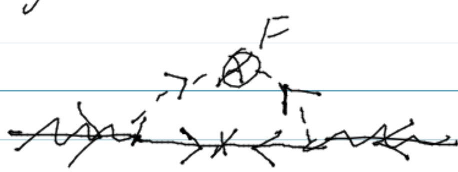
$$\sqrt{\langle F \rangle} \sim 10^{10} \text{ to } 10^{11} \text{ GeV}$$

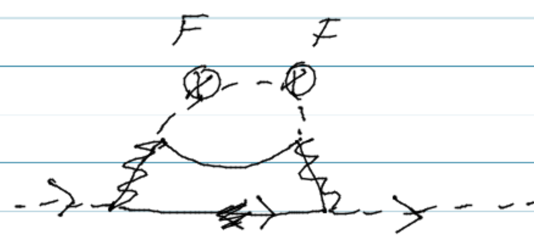
$$F_{comp} \propto \lambda\lambda \sim \Lambda^3$$

$$m_{soft} \sim \frac{\Lambda^3}{M_{pl}^2} \qquad \langle F_{comp} \rangle = \frac{\Lambda^3}{M_{pl}}$$

$$\Lambda \doteq 10^{13} \text{ GeV}$$

"gauge mediation"





$$m_{soft} \sim \frac{\alpha_a}{4\pi} \frac{\langle F_{mess} \rangle}{M_{mess}}$$

$\sqrt{F_{mess}}$ as low as 10, 100 TeV