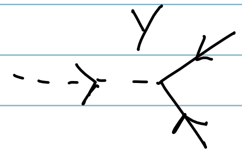
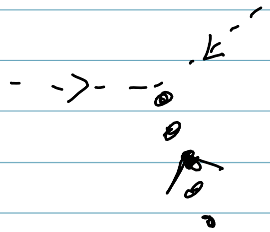
Superpotentials

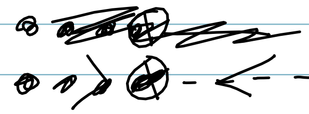
Y

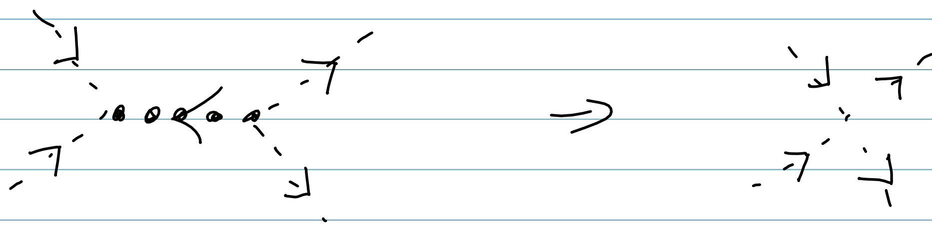


M

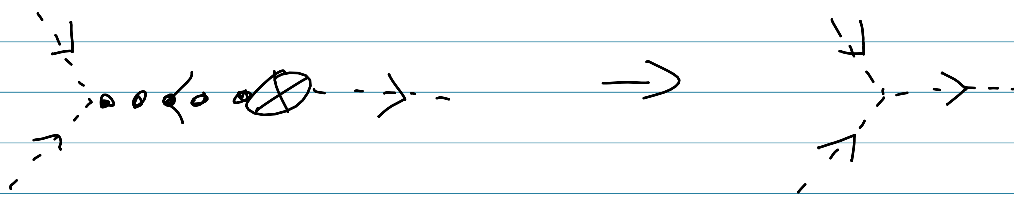
helicity flip



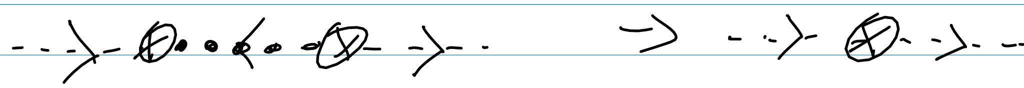




YY^*



YM^*



no "helicity" flip

MM^*

super current:

$$J^\mu_\alpha = (\sigma^\nu \bar\sigma^\mu \psi_i)_\alpha D_\nu \phi^{*i} - i(\sigma^\mu \psi^{\dagger i})_\alpha W^*_i$$

$$- \frac{1}{2\sqrt{2}} (\sigma^\nu \bar\sigma^\rho \sigma^\mu \lambda^{\dagger a})_\alpha F^a_{\nu\rho} - \frac{i}{\sqrt{2}} g \phi^* T^a \phi (\sigma^\mu \lambda^{\dagger a})_\alpha$$

$$= \frac{i}{\sqrt{2}} D^a (\sigma^\mu \lambda^{\dagger a})_\alpha + F_j\, i (\sigma^\mu \psi^{\dagger j})_\alpha$$

$$+ (\sigma^\nu \bar\sigma^\mu \psi_j)_\alpha D_\nu \phi^* - \frac{1}{2\sqrt{2}} (\sigma^\nu \bar\sigma^\rho \sigma^\mu \lambda^{\dagger a})_\alpha \; {}^{a}$$

Superspace   no new information

Introduce anticommuting (Grassman) spinors
θ_α, $\bar{\theta}_{\dot{\alpha}}$

recall $\int d\theta = 0$ $\int \theta \, d\theta = 1$

$$\epsilon_{\alpha\beta} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \qquad \epsilon^{\alpha\beta} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$d^2\theta \equiv -\frac{1}{4} d\theta^\alpha d\theta^\beta \epsilon_{\alpha\beta}$$

$$d^2\bar{\theta} \equiv -\frac{1}{4} d\bar{\theta}_{\dot{\alpha}} d\bar{\theta}_{\dot{\beta}} \epsilon^{\dot{\alpha}\dot{\beta}}$$

$$d^4\theta \equiv d^2\theta \, d^2\bar{\theta}$$

$$\int d^2\theta \, \theta^2 = -\frac{1}{4} \int d\theta^\alpha d\theta^\beta \epsilon_{\alpha\beta} \, \theta^\sigma \epsilon_{\sigma\tau} \theta^\tau$$

$$= -\frac{1}{4} \left(\epsilon_{\alpha\beta} \delta^{\beta\sigma} \epsilon_{\sigma\tau} \delta^{\alpha\tau} - \epsilon_{\alpha\beta} \delta^{\sigma\alpha} \epsilon_{\sigma\tau} \delta^{\tau\beta} \right)$$

$$= -\frac{1}{4} \left(\epsilon_{\alpha\beta} \epsilon_{\beta\alpha} + \epsilon_{\alpha\beta} \epsilon_{\beta\alpha} \right) = -\frac{1}{2} \epsilon_{\alpha\beta} \epsilon_{\beta\alpha}$$

$$= 1$$

$$\int d^2\theta \, (\chi\theta)(\theta\psi) = -\frac{1}{4} \int d\theta^\alpha d\theta^\beta \epsilon_{\alpha\beta} \chi^\sigma \epsilon_{\sigma\tau} \theta^\tau \theta^\gamma \epsilon_{\gamma\delta} \psi^\delta$$

$$= -\frac{1}{4} \left(\epsilon_{\alpha\beta} \chi^\sigma \epsilon_{\sigma\beta} \epsilon_{\alpha\delta} \psi^\delta - \epsilon_{\alpha\beta} \chi^\sigma \epsilon_{\sigma\alpha} \epsilon_{\beta\delta} \psi^\delta \right)$$

$$= -\frac{1}{4} \left(-\chi^\sigma \epsilon_{\sigma\beta} \epsilon_{\beta\alpha} \epsilon_{\alpha\delta} \psi^\delta - \chi^\sigma \epsilon_{\sigma\alpha} \epsilon_{\alpha\beta} \epsilon_{\beta\delta} \psi^\delta \right)$$

$$= -\frac{2}{4} \left(\chi^\sigma \epsilon_{\sigma\delta} \psi^\delta \right. = -\frac{1}{2} (\chi\psi)$$

$$\chi_\alpha (\xi\eta) = -\xi_\alpha (\chi\eta) - (\xi\chi)\eta_\alpha$$

$$(\chi\theta)(\theta\psi) = \chi^\alpha \left(-\theta_\alpha (\theta\psi) - (\theta\theta)\psi_\alpha \right)$$

$$= -(\chi\theta)(\theta\psi) - (\theta\theta)(\chi\psi)$$

$$2(\chi\theta)(\theta\psi) = -(\theta\theta)(\chi\psi)$$

$$(\chi\theta)(\theta\psi) = -\frac{1}{2} (\theta\theta)(\chi\psi)$$

$$\psi^\dagger \bar\sigma^\mu \chi = -\chi \sigma^\mu \psi^\dagger \qquad (\theta\sigma^\mu\bar\theta)^2 = -\theta\sigma^\mu\bar\theta\;\theta\sigma^\nu\bar\theta$$
$$= \tfrac{\cancel{+}}{2}\,\theta\theta\,(\bar\theta\bar\sigma^\mu\sigma^\nu\bar\theta$$

$$y^\mu = x^\mu - i\theta\sigma^\mu\bar\theta$$

$$\Phi = \phi(y) + \sqrt{2}\,\theta\psi(y) + \theta^2 F(y)$$

$$= \phi(x) + \partial_\mu\phi(-i\theta\sigma^\mu\bar\theta) + \tfrac{1}{2}\partial_\mu\partial_\nu\phi(i\theta\sigma^\mu\bar\theta)(i\theta\sigma^\nu\bar\theta)$$
$$+ \sqrt{2}\,\theta\psi + \sqrt{2}\,\theta\,\partial_\mu\psi(-i\theta\sigma^\mu\bar\theta) + \theta^2 F$$

$$= \phi(x) - i\theta\sigma^\mu\bar\theta\,\partial_\mu\phi - \tfrac{1}{2}(+\tfrac{1}{2})\,\theta^2\bar\theta^2\,\partial^2\phi$$

$$+ \sqrt{2}\,\theta\psi + \frac{i\sqrt{2}}{2}\,\theta^2\,\partial_\mu\psi\sigma^\mu\bar\theta + \theta^2 F$$

$$\int d^4\theta\,\Phi^\dagger\Phi = \int d^4\theta\,\Big(\phi^* + i\theta\sigma^\mu\bar\theta\,\partial_\mu\phi^* - \tfrac{1}{4}\theta^2\bar\theta^2\partial^2\phi^*$$
$$+ \sqrt{2}\,\bar\theta\psi^\dagger - \tfrac{i}{\sqrt{2}}\,\bar\theta^2\,\theta\sigma^\mu\partial_\mu\psi^\dagger + \bar\theta^2 F^*\Big)$$
$$\times\Big(\phi - i\theta\sigma^\mu\bar\theta\,\partial_\mu\phi - \tfrac{1}{4}\theta^2\bar\theta^2\partial^2\phi$$
$$+ \sqrt{2}\,\theta\psi + \tfrac{i}{\sqrt{2}}\,\theta^2\partial_\nu\psi\sigma^\nu\bar\theta + \theta^2 F\Big)$$
$$= F^*F - \tfrac{1}{4}(\phi^*\partial^2\phi + \partial^2\phi^*\phi)$$

$$+ \int d^4\theta\,(\theta\sigma^\mu\bar\theta)(\theta\sigma^\nu\bar\theta)\,\partial_\mu\phi^*\partial_\nu\phi$$
$$+ i\int d^4\theta\,(\bar\theta\psi^\dagger)(\theta^2\partial_\mu\psi\sigma^\mu\bar\theta) - (\bar\theta^2\theta\sigma^\mu\partial_\mu\psi^\dagger)(\theta\psi)$$

$$= F^*F - \tfrac{1}{4}\Big(\partial^\mu(\phi^*\partial_\mu\phi) - \partial^\mu\phi^*\partial_\mu\phi$$
$$\partial^\mu(\partial_\mu\phi^*\phi) - \partial^\mu\phi^*\partial_\mu\phi\Big)$$
$$+ \tfrac{1}{2}\,\partial^\mu\phi^*\partial_\nu\phi$$

$$- \tfrac{i}{2}\int d^4\theta\;\bar\theta^2(\theta^2\partial_\mu\psi\sigma^\mu\psi^\dagger) - \bar\theta\psi\sigma^\mu\partial_\mu\psi^\dagger\theta^2$$

$$= F^*F - \tfrac{1}{4}\partial^\mu(\phi^*\partial_\mu\phi + \partial_\mu\phi^*\phi) + \partial^\mu\phi^*\partial_\mu\phi$$

$$- \tfrac{i}{2}\big(\partial_\mu\psi\sigma^\mu\psi^\dagger - \partial_\mu(\psi\sigma^\mu\psi^\dagger) + \partial_\mu\psi\sigma^\mu\psi^\dagger\big)$$

$$= F^*F + \partial^\mu\phi^*\partial_\mu\phi + i\psi^\dagger\bar\sigma^\mu\partial_\mu\psi$$
$$- \tfrac{1}{4}\partial^\mu(\phi^*\partial_\mu\phi + \partial_\mu\phi^*\phi) + \tfrac{i}{2}\partial_\mu(\psi\sigma^\mu\psi^\dagger)$$

$$\int d^4x\int d^4\theta\,\Phi^\dagger\Phi = \int d^4x\;\mathcal{L}_{WZ}$$

$$\int d^2\theta\, W(\Phi) = \int d^2\theta\, W|_{\theta=0} + \theta W_1 + \theta^2 W_2 = W_2$$

$$\int d^2\theta \left(\frac{\delta W}{\delta \phi_i}\Big|_{\theta=0} \left(-\frac{1}{4}\theta^2\bar\theta^2 \partial^2 \phi_i + \frac{i}{\sqrt2}\theta^2 \partial_\mu \psi_i \sigma^\mu \bar\theta + \theta^2 F_i \right) + \frac{1}{2} \frac{\delta W}{\delta\phi_i \delta\phi_j}\Big|_{\theta=0} \left(-i\theta\sigma^\mu\bar\theta \partial_\mu \phi + \sqrt2 \theta \psi_i\right) \times \left(-i\theta\sigma^\nu\bar\theta\partial_\mu \phi_j + \sqrt2\theta\psi_j\right) \right)$$

$$= W^i \left(F_i - \frac14 \bar\theta^2 \partial^2 \phi_i + \frac{i}{\sqrt2} \partial_\mu \psi_i \sigma^\mu \bar\theta \right)$$

$$+ \int d^2\theta \frac12 W^{ij} \begin{pmatrix} 2(\theta\psi_i)(\theta\psi_j) - i\sqrt2 \begin{pmatrix} (\theta\psi_i)(\theta\sigma^\nu\bar\theta\partial_\mu\phi_j) \\ (\theta\sigma^\mu\bar\theta\partial_\mu\phi_i)(\theta\psi_j) \end{pmatrix} \\ -(\theta\sigma^\mu\bar\theta\cancel{\partial_\mu}\phi)(\theta\sigma^\nu\bar\theta)\partial_\mu\phi_i\partial_\nu\phi_j \end{pmatrix}$$

$$= W^i F_i - \frac14 \partial^\mu (W^i \bar\theta^2 \partial_\mu \phi_i) + \frac14 (\partial_\mu W^i)(\bar\theta \partial^\mu \phi_i) + \frac{i}{\sqrt2}\partial^\mu (W^i \psi \sigma^\mu \bar\theta) - \frac{i}{\sqrt2}(\partial_\mu W^i)(\psi_i \sigma^\mu \bar\theta)$$

$$+ \frac12 W^{ij} \left(-\psi_i\psi_j + \frac{i}{\sqrt2} \begin{pmatrix} \psi_i\sigma^\mu\bar\theta\partial_\mu\phi_j \\ + \psi_j\sigma^\mu\bar\theta\partial_\mu\phi_i \end{pmatrix} \frac12 \eta^{\mu\nu}\bar\theta^2\partial_\mu\phi_i\partial_\nu\phi_j \right)$$

$\partial_\mu W^i = W^{ij}\partial_\mu\phi_j$

$$= W^i F_i - \frac14 \partial^\mu (W^i \bar\theta \partial_\mu \phi_i) + \frac{i}{\sqrt2}\partial^\mu (W^i \psi \sigma^\mu \bar\theta) + (\partial^\mu W^i)\left(\frac14 \bar\theta^2 \partial^\mu \phi - \frac{i}{\sqrt2}\psi_i \sigma^\nu\bar\theta\right)$$

$$-\frac12 W^{ij}\psi_i\psi_j + \frac12 (\partial_\mu W^i)\left(\frac{i}{\sqrt2} 2\psi\sigma^\mu\bar\theta - \frac12 \bar\theta^2 \partial^\mu \phi\right)$$

$$= W^i F_i + \partial^\mu \left(-\frac14 W^i \bar\theta^2 \partial_\mu \phi_i + \frac{i}{\sqrt2} W^i \psi_i \sigma^\mu \bar\theta\right) - \frac12 W^{ij}\psi_i\psi_j$$

$$\int d^4x\, d^2\theta\, W(\Phi) = \int d^4x\, \mathcal{L}_{WZint}$$

chiral × chiral = chiral'

$\delta_{susy} W|_{\theta^2} = \partial_\mu V^\mu$

$\Rightarrow \int d^4x\, d^2\theta\, W$ is a SUSY invariant

most general int $\int d^4x\, d^4\theta\, K(\Phi^\dagger, \Phi)$

K ← real

$$\delta_{susy} K|_{\theta^2\bar\theta^2} = \partial_\mu V^\mu$$

Vector supermultiplet as a real superfield

$A^a_\mu,\ \lambda^a_\alpha,\ D^a,\ \underbrace{\phi^a_{1,2,3}\ \ \psi^a_\alpha}$

3   4   1   real 3   4

W-Z gauge removes Λ complex

$$V^a = \theta\sigma^\mu\bar\theta A^a_\mu + \theta^2\bar\theta\lambda^{\dagger a} + \bar\theta^2\theta\lambda^a + \frac{1}{2}\theta^2\bar\theta^2 D$$

$$V^a V^b = \frac{1}{2}\theta^2\bar\theta^2 A^a_\mu A^b_\mu$$

$$V^a V^b V^c = 0$$

chiral superfield

$$e^{T^a V^a} \underset{gauge}{\rightarrow} e^{T^a\Lambda^{a\dagger}} e^{T^a V^a} e^{T^a\Lambda^a}$$

$$V^a \rightarrow V^a + \Lambda^a + \Lambda^{a\dagger} + \mathcal{O}(V^a\Lambda^b)$$

$$\Phi \rightarrow e^{-gT^a\Lambda^a}\Phi$$

$$D_\alpha = \frac{\partial}{\partial \theta^\alpha} - 2i(\sigma^\mu \bar{\theta})_\alpha \frac{\partial}{\partial y^\mu}$$

$$\bar{D}_{\dot{\alpha}} = -\frac{\partial}{\partial \bar{\theta}}$$

$$T^a W^a_\alpha = -\frac{1}{4} \bar{D}_{\dot{\alpha}} \bar{D}^{\dot{\alpha}} e^{-T^a V^a} D_\alpha e^{T^a V^a}$$

$$W^a_\alpha = -i\lambda_\alpha(y) + \theta_\alpha D^a_{(y)} - (\sigma^{\mu\nu}\theta_\alpha) F^a_{\mu\nu}(x) - \theta\theta\sigma^\mu D_\mu \lambda^{+a}_{(y)}$$

$$\bar{D} W_\alpha = 0 \qquad \sigma^{\mu\nu} = \frac{1}{4}(\sigma^\mu \bar{\sigma}^\nu - \sigma^\nu \bar{\sigma}^\mu)$$

$$\int d^4x \, \mathcal{L}_{sym} = \frac{1}{4} \int d^4x \, d^2\theta \, W^{a\alpha} W^a_\alpha + h.c.$$

$$= \frac{1}{4} \int \cancel{d^4x} \, d^4\theta \, \mathrm{Tr}\, T^a W^{a\alpha} e^{-T^a V^a} D_\alpha e^{T^a V^a} + h.c.$$

$$\int d^4\theta \, \Phi^+ e^{gT^a V^a} \Phi$$

$$\int d^4\theta \, K(\Phi^+, e^{gT^a V^a} \Phi)$$