

$$Y_{im, a}^{jm} = \sqrt{2g} (T^a)^n_m \int_i^j = \begin{matrix} i & j, n \\ & \nearrow \searrow \\ i, m & \rightarrow & \text{wavy line} & a \end{matrix}$$

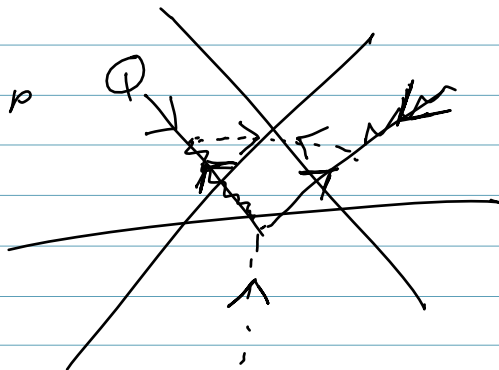
$$Y_2(\phi) = \begin{matrix} \phi \\ \vdots \\ \rightarrow \text{wavy line} \rightarrow \\ \Phi \quad \lambda \quad \Phi \end{matrix} = \sqrt{2g} T^a_n \sqrt{2g} T^a_k$$

$$\Rightarrow 2g^2 \text{ [wavy line diagram]} \\ = 2g^2 C_2(\square) \int_m^k$$

$$Y_2(\lambda) = \begin{matrix} \phi \\ \vdots \\ \leftarrow \text{wavy line} \leftarrow \\ \lambda^a \quad \Phi \quad \lambda^b \end{matrix}$$

$$\rightarrow 2g^2 \text{ [circle diagram]} \\ = 2g^2 2FT(\square) \int^{ab} \\ = 2g^2 F \int^{ab}$$

$$Y^p Y^{k\dagger} Y^p$$



$$\text{Tr } Y^\dagger Y \rightarrow \text{[circle diagram]} \rightarrow 2g^2 \text{ [wavy line diagram]} \\ = 2g^2 C_2(\square)$$

$$\{ C_2(F), Y \} = \Phi \begin{matrix} \text{wavy line} \\ \vdots \\ \Phi \end{matrix} + \Phi \begin{matrix} \text{wavy line} \\ \vdots \\ Y \end{matrix}$$

$$= C_2(\square) \sqrt{2g} T^a + C_2(\text{Ad}) \sqrt{2g} T^a$$

$$\begin{aligned}
(4\pi)^2 \beta_Y &= \frac{1}{2} (2g^2 C_2(\square) \sqrt{2}g + \sqrt{2}g 2g^2 F) \\
&+ 2g^2 C_2(\square) \sqrt{2}g \\
&- 3g^2 (C_2(\square) \sqrt{2}g + \sqrt{2}g N) \\
&= \sqrt{2}g^3 (C_2(\square) + F + 2C_2(\square)) \\
&\quad (-3C_2(\square) - 3N) \\
&= -\sqrt{2}g^3 (3N - F) \\
&= \sqrt{2} (4\pi)^2 \beta_g
\end{aligned}$$

The D-term coupling
quartic $\lambda = g^2$?

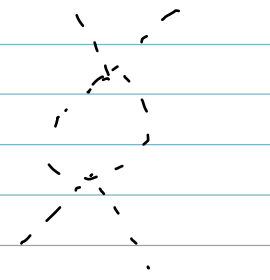
$$\begin{aligned}
D^a &= g (\phi^{*in} T^a)_n \phi_{mi} - \bar{\phi}^{ih} (T^a)_n \bar{\phi}_{mi}^* \\
V &= \frac{1}{2} D^a D^a = \frac{g^2}{2} (\phi^{*i} T^a \phi_i - \bar{\phi}^i T^a \bar{\phi}_i^*) (\phi^{*j} T^a \phi_j - \bar{\phi}^j T^a \bar{\phi}_j^*)
\end{aligned}$$

$$= -ig^2 (T^a)_n^m (T^a)_l^p$$

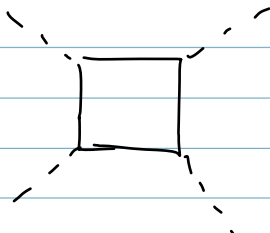
$$= -ig^2 \left((T^a)_n^m (T^a)_l^p + (T^a)_l^m (T^a)_n^p \right)$$

$$(47^2) \beta_1 = \Lambda^2 - 4H + 3A + \Lambda^4 - 3\Lambda^5$$

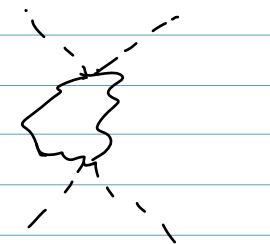
$\Lambda^2 \sim$



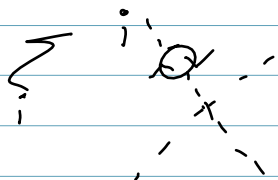
$H \sim$



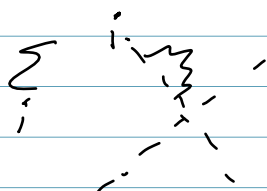
$A \sim$



$\Lambda^4 \sim$

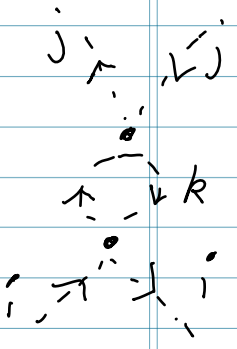


$\Lambda^5 \sim$

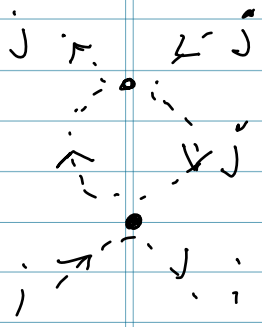


} 47^2

$$\begin{array}{c} i \\ \nearrow \\ \bullet \\ \searrow \\ j \end{array} \rightarrow \begin{array}{c} \leftarrow \\ \text{---} \\ \rightarrow \end{array} = \frac{g^2}{2} \left(\begin{array}{c} \leftarrow \\ \text{---} \\ \rightarrow \end{array} - \frac{1}{N} \begin{array}{c} \leftarrow \\ \text{---} \\ \rightarrow \end{array} \right)$$



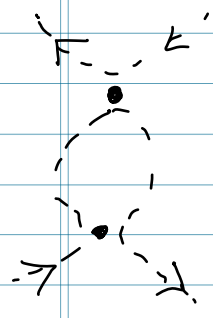
$$\Rightarrow g^4 2F \begin{array}{c} \leftarrow \\ \text{---} \\ \bullet \\ \text{---} \\ \rightarrow \end{array} = g^4 2FT(\square) \begin{array}{c} \leftarrow \\ \text{---} \\ \rightarrow \end{array} = g^4 F \begin{array}{c} \leftarrow \\ \text{---} \\ \rightarrow \end{array}$$



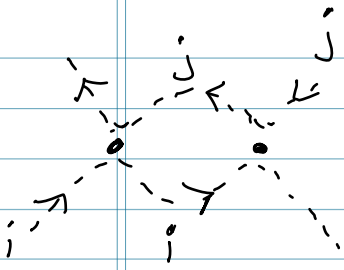
$$\Rightarrow g^4 \begin{array}{c} \leftarrow \\ \text{---} \\ \bullet \\ \text{---} \\ \rightarrow \end{array} = \frac{g^4}{4} \left(\begin{array}{c} \leftarrow \\ \text{---} \\ \rightarrow \end{array} - \frac{1}{N} \left(\begin{array}{c} \leftarrow \\ \text{---} \\ \rightarrow \end{array} + \frac{1}{N^2} \begin{array}{c} \leftarrow \\ \text{---} \\ \rightarrow \end{array} \right) \right)$$

$$= \frac{g^4}{4} \left(\begin{array}{c} \leftarrow \\ \text{---} \\ \rightarrow \end{array} - \frac{1}{N} \begin{array}{c} \leftarrow \\ \text{---} \\ \rightarrow \end{array} + \frac{1}{N^2} \begin{array}{c} \leftarrow \\ \text{---} \\ \rightarrow \end{array} \right)$$

$$= -\frac{g^4}{4N} \left(\begin{array}{c} \leftarrow \\ \text{---} \\ \rightarrow \end{array} - \frac{1}{N} \begin{array}{c} \leftarrow \\ \text{---} \\ \rightarrow \end{array} \right) = -\frac{g^4}{2N} \begin{array}{c} \leftarrow \\ \text{---} \\ \rightarrow \end{array}$$



$$\Rightarrow g^4 \begin{array}{c} \leftarrow \\ \text{---} \\ \bullet \\ \text{---} \\ \rightarrow \end{array} = -\frac{g^4}{2N} \begin{array}{c} \leftarrow \\ \text{---} \\ \rightarrow \end{array}$$

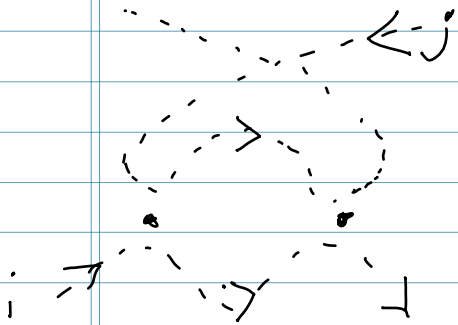


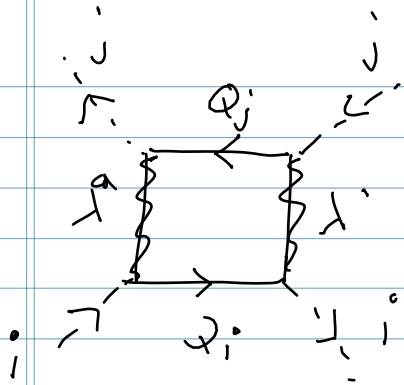
$$\rightarrow g^4 \left[\text{Diagram} \right] = \frac{g^4}{4} \left(\left[\text{Diagram 1} \right] - \frac{1}{N} \left[\text{Diagram 2} \right] + \frac{1}{N^2} \left[\text{Diagram 3} \right] \right)$$

$$= \frac{g^4}{4} \left(N \uparrow \downarrow - \frac{2}{N} \uparrow \downarrow + \frac{1}{N^2} \left[\text{Diagram 3} \right] \right)$$

$$= \frac{g^4}{4} \left(\frac{N^2 - 2}{N} \left(2 \left[\text{Diagram 3} \right] + \frac{1}{N} \left[\text{Diagram 3} \right] \right) + \frac{1}{N^2} \left[\text{Diagram 3} \right] \right)$$

$$= g^4 \left(\frac{N^2 - 2}{2N} \left[\text{Diagram 3} \right] + \frac{1}{4} \left(1 - \frac{1}{N^2} \right) \left[\text{Diagram 3} \right] \right)$$

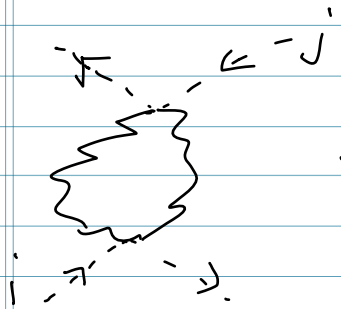




$$\rightarrow (\sqrt{2}g)^4 \begin{array}{c} \leftarrow \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \rightarrow \end{array}$$

$$= 4g^4 \left(\frac{1}{2} \left(\frac{N^2-2}{N} \right) \begin{array}{c} \leftarrow \\ \text{---} \\ \text{---} \\ \text{---} \\ \rightarrow \end{array} + \frac{1}{4} \left(1 - \frac{1}{N^2} \right) \begin{array}{c} \leftarrow \\ \text{---} \\ \text{---} \\ \text{---} \\ \rightarrow \end{array} \right)$$

$$= g^4 \left(2 \left(\frac{N^2-2}{N} \right) \begin{array}{c} \leftarrow \\ \text{---} \\ \text{---} \\ \text{---} \\ \rightarrow \end{array} + \left(1 - \frac{1}{N^2} \right) \begin{array}{c} \leftarrow \\ \text{---} \\ \text{---} \\ \text{---} \\ \rightarrow \end{array} \right)$$



$$\rightarrow g^4 2 \left(\begin{array}{c} \leftarrow \\ \text{---} \\ \text{---} \\ \text{---} \\ \rightarrow \end{array} + \begin{array}{c} \leftarrow \\ \text{---} \\ \text{---} \\ \text{---} \\ \rightarrow \end{array} \right)$$

$$= g^4 2 \left(\frac{1}{2} \left(\frac{N^2-2}{N} \right) \begin{array}{c} \leftarrow \\ \text{---} \\ \text{---} \\ \text{---} \\ \rightarrow \end{array} + \frac{1}{4} \left(1 - \frac{1}{N^2} \right) \begin{array}{c} \leftarrow \\ \text{---} \\ \text{---} \\ \text{---} \\ \rightarrow \end{array} \right.$$

$$\left. + \frac{1}{N} \begin{array}{c} \leftarrow \\ \text{---} \\ \text{---} \\ \text{---} \\ \rightarrow \end{array} + \frac{1}{4} \left(1 - \frac{1}{N^2} \right) \begin{array}{c} \leftarrow \\ \text{---} \\ \text{---} \\ \text{---} \\ \rightarrow \end{array} \right)$$

$$= g^4 \left(\frac{N^2-4}{N} \begin{array}{c} \leftarrow \\ \text{---} \\ \text{---} \\ \text{---} \\ \rightarrow \end{array} + \left(1 - \frac{1}{N^2} \right) \begin{array}{c} \leftarrow \\ \text{---} \\ \text{---} \\ \text{---} \\ \rightarrow \end{array} \right)$$

Factor of 2

$$\Lambda^2 = \text{[Diagram: tadpole with a square loop] } = \sum_{\leftarrow \rightarrow} g^4 \left(2F - \frac{2}{N} + \frac{N^2 - 2}{N} - \frac{2}{N} \right) + \left(1 - \frac{1}{N^2} \right) \sum_{\leftarrow \rightarrow}$$

$$-4H = -8g^4 \left(\frac{N^2 - 2}{N} \right) \sum_{\leftarrow \rightarrow} -4g^4 \left(1 - \frac{1}{N^2} \right) \sum_{\leftarrow \rightarrow}$$

$$3A = \text{[Diagram: tadpole with a triangle loop] } = 3g^4 \left(\frac{N^2 - 4}{N} \right) \sum_{\leftarrow \rightarrow} + 3g^4 \left(1 - \frac{1}{N^2} \right) \sum_{\leftarrow \rightarrow}$$

$$N^2 = \text{[Diagram: tadpole with a square loop and a vertex] } = 4(\sqrt{2}g)^2 C_2(\square) g^2 \sum_{\leftarrow \rightarrow} = 4g^4 \left(\frac{N^2 - 1}{N} \right) \sum_{\leftarrow \rightarrow}$$

$$-3A^S = -3 \text{[Diagram: tadpole with a square loop and a vertex] } = -3 \cdot 4g C_2(\square) g^2 \sum_{\leftarrow \rightarrow} = -6g^4 \left(\frac{N^2 - 1}{N} \right) \sum_{\leftarrow \rightarrow}$$

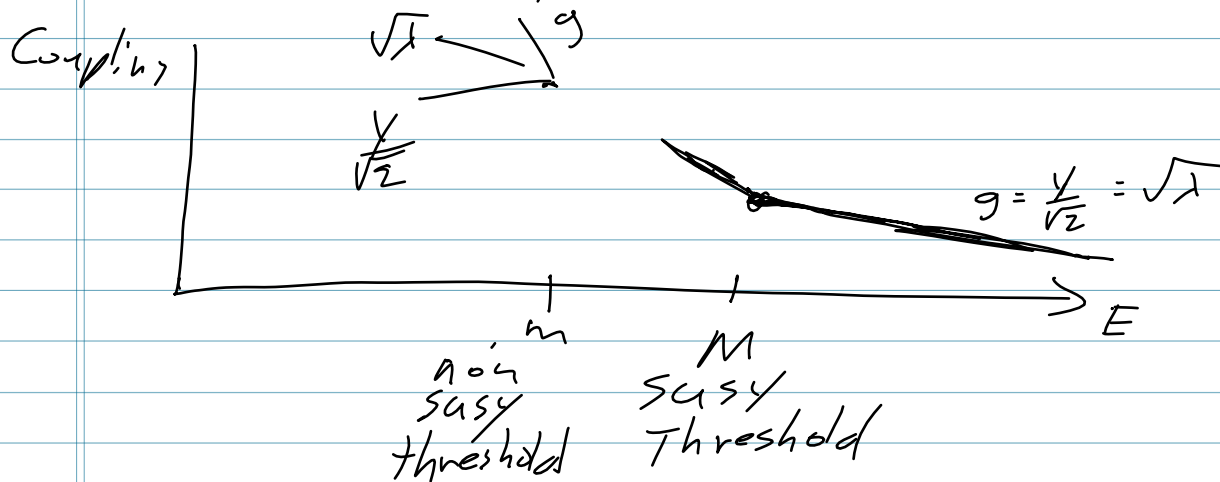
$$(4\pi)^2 \beta_\lambda = \sum_{\leftarrow \rightarrow} g^4 \left(2F + N - \frac{6}{N} - 8N + \frac{16}{N} + \frac{3N - 12}{N} + 4N - \frac{4}{N} - 6N + \frac{6}{N} \right)$$

$$+ \sum_{\leftarrow \rightarrow} g^4 \left(1 - \frac{1}{N^2} + 4 \left(1 - \frac{1}{N^2} \right) + 3 \left(1 - \frac{1}{N^2} \right) \right) = \sum_{\leftarrow \rightarrow} g^4 (2F - 6N)$$

$$\beta_\lambda = \beta_{g^2} = 2g \beta_g$$

SUSY is not anomalous at one-loop

Add SUSY masses and SUSY breaking masses:



scalar mass will have quad div.

$$\propto m^2$$

this is why we can't dim 4
SUSY breaking terms

without re-introducing quad. div.

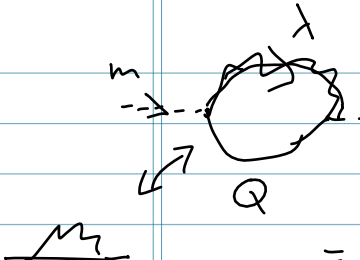
quad divergences from gauge interactions

$$\rightarrow \text{tadpole} = 0 \quad \text{since } T_n^a = 0$$

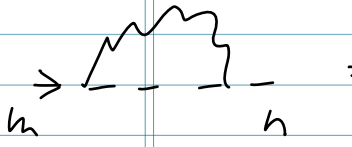
$$\rightarrow \text{self-energy} = -ig^2 T_n^a T_p^a \int \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2}$$

$$= -\frac{ig^2 C_2(\square) d_n}{16\pi^2} \int_0^{M^2} k^2 \frac{dk^3}{k^2}$$

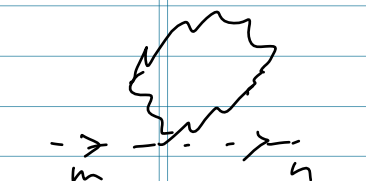
$$T^a(\phi^\dagger \epsilon^{\alpha\beta} \psi_\beta \lambda_\alpha + \phi \epsilon^{\alpha\beta} \lambda_\alpha^\dagger \psi_\beta^\dagger)$$



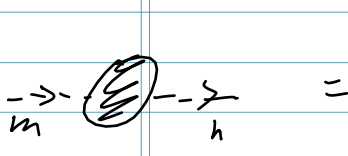
$$\begin{aligned}
 & (-i\sqrt{2}g)^2 T_n^a T_p^a (-1) \int \frac{d^4 k}{(2\pi)^4} \text{Tr} \frac{i \epsilon}{\not{k}} \frac{i \epsilon^T}{\not{k}} \\
 &= -2g^2 \frac{C_2(\square)}{16\pi^2} \int_0^1 dk^2 \text{Tr} \frac{\not{k} \epsilon \not{k} \epsilon^T}{k^4} \\
 &= 4ig^2 \frac{C_2(\square)}{16\pi^2} \int_0^1 dk^2
 \end{aligned}$$



$$\begin{aligned}
 &= (ig^2) C_2(\square) \int \frac{d^4 k}{(2\pi)^4} \frac{i \not{k} (-i) (g_{\mu\nu} + (\xi-1) \frac{k_\mu k_\nu}{k^2})}{k^2} \\
 &= -g^2 C_2(\square) \int \frac{d^4 k}{(2\pi)^4} \frac{(k^2 + (\xi-1)k^2)}{k^4} \\
 &= ig^2 \xi \frac{C_2(\square)}{16\pi^2} \int_0^1 dk^2
 \end{aligned}$$



$$\begin{aligned}
 &= ig^2 \left(\frac{1}{2}\right) (T^a T^b + T^b T^a) \delta g^{\mu\nu} \int \frac{d^4 k}{(2\pi)^4} \frac{(-i) (g_{\mu\nu} + (\xi-1) \frac{k_\mu k_\nu}{k^2})}{k^2} \\
 &= g^2 C_2(\square) \int \frac{d^4 k}{(2\pi)^4} \frac{(4 + (\xi-1))}{k^2} \\
 &= -\frac{ig^2}{16\pi^2} C_2(\square) \int_0^1 dk^2 (3+\xi)
 \end{aligned}$$



$$\begin{aligned}
 &= (-1 + 4 + \xi - (3+\xi)) \frac{ig^2}{16\pi^2} C_2(\square) \int_0^1 dk^2 \\
 &= 0
 \end{aligned}$$